Задача 1 (*1 балл*)

Для заданных функций φ и ψ:

а) построить графики функций φ и ψ;

б) вычислить свертку $\varphi * \psi$ функций φ и ψ;

в) построить график свертки $\varphi * \psi$.

Функция φ задана формулой, график функции ψ — ломаная, соединяющая точки $A(x_1,\ y_1)$, $B(x_2,\ y_2)$, $C(x_3,\ y_3)$, $D(x_4,\ y_4)$ (вне отрезка $[x_1,\ x_4]$ функция равна нулю).

Вар.	$\varphi(x)$	A	B	C	D
1	$\operatorname{rect} x$	$(-2,1)$	$(-1,-1)$	$(1,-1)$	$(2,1)$
2	$\operatorname{rect}\frac{x}{2}$	$(-2,1)$	$(0,-1)$	$(1,-1)$	$(3,1)$
3	$\operatorname{rect} x$	$(-2,0)$	$(-1,2)$	$(1,2)$	$(2,0)$
4	$\operatorname{rect}\frac{x}{2}$	$(-2,0)$	$(0,2)$	$(1,2)$	$(3,0)$
5	$\operatorname{rect} x$	$(-2,-2)$	$(-1,0)$	$(1,0)$	$(2,2)$
6	$\operatorname{rect} x$	$(-2,2)$	$(-1,0)$	$(1,0)$	$(2,-2)$
7	$\operatorname{rect} x$	$(0,0)$	$(1,-2)$	$(3,-2)$	$(4,0)$
8	$\operatorname{rect}\frac{x}{2}$	$(-2,1)$	$(-1,-1)$	$(1,-1)$	$(2,1)$
9	$\operatorname{rect} x$	$(0,2)$	$(2,0)$	$(3,0)$	$(5,2)$
10	$\operatorname{rect} x$	$(0,-1)$	$(1,1)$	$(3,1)$	$(4,-1)$
11	$\operatorname{rect}\frac{x}{2}$	$(0,2)$	$(2,0)$	$(3,0)$	$(5,2)$
12	$\operatorname{rect}\frac{x}{2}$	$(0,-1)$	$(1,1)$	$(3,1)$	$(4,-1)$
13	$\operatorname{rect}(x-1)$	$(-2,1)$	$(-1,-1)$	$(1,-1)$	$(2,1)$
14	$\operatorname{rect}(x-1)$	$(-2,1)$	$(-1,-1)$	$(1,-1)$	$(2,1)$
15	$\operatorname{rect} x$	$(-2,1)$	$(0,-1)$	$(1,-1)$	$(3,1)$

Вар.	$\varphi(x)$	A	B	C	D
16	$-\operatorname{rect} x$	$(-2,0)$	$(-1,2)$	$(1,2)$	$(2,0)$
17	$-\operatorname{rect}\frac{x}{2}$	$(-2,0)$	$(0,2)$	$(1,2)$	$(3,0)$
18	$-\operatorname{rect} x$	$(-2,-2)$	$(-1,0)$	$(1,0)$	$(2,2)$
19	$-\operatorname{rect} x$	$(-2,2)$	$(-1,0)$	$(1,0)$	$(2,-2)$
20	$-\operatorname{rect} x$	$(0,0)$	$(1,-2)$	$(3,-2)$	$(4,0)$
21	$-\operatorname{rect}\frac{x}{2}$	$(-2,1)$	$(-1,-1)$	$(1,-1)$	$(2,1)$
22	$-\operatorname{rect} x$	$(0,2)$	$(2,0)$	$(3,0)$	$(5,2)$
23	$-\operatorname{rect} x$	$(0,-1)$	$(1,1)$	$(3,1)$	$(4,-1)$
24	$-\operatorname{rect}\frac{x}{2}$	$(0,2)$	$(2,0)$	$(3,0)$	$(5,2)$
25	$-\operatorname{rect}\frac{x}{2}$	$(0,-1)$	$(1,1)$	$(3,1)$	$(4,-1)$
26	$-\operatorname{rect}(x-1)$	$(-2,1)$	$(-1,-1)$	$(1,-1)$	$(2,1)$
27	$-\operatorname{rect} x$	$(-2,1)$	$(-1,-1)$	$(1,-1)$	$(2,1)$
28	$-\operatorname{rect}\frac{x}{2}$	$(-2,1)$	$(0,-1)$	$(1,-1)$	$(3,1)$
29	$-\operatorname{rect} x$	$(-2,1)$	$(-1,-1)$	$(1,-1)$	$(2,1)$
30	$-\operatorname{rect}\frac{x}{2}$	$(-2,1)$	$(0,-1)$	$(1,-1)$	$(3,1)$

Примечание.

$$\operatorname{rect} x = \eta\Big(\frac{1}{2} - |x|\Big) = \begin{cases} 1, & |x| < \frac{1}{2}; \\ 0, & |x| > \frac{1}{2}. \end{cases}$$

Задача 2 (*1 балл*)

Определите тип дифференциального уравнения, приведите его к каноническому виду, запишите общее решение, найдите решение задачи Коши.

1. $u_{xx} - 2x\,u_{xy} + x^2\,u_{yy} - u_y = 0;\quad u(0,y) = y^2,\ u_x(0,y) = y.$

2. $u_{xx} + 2(\sin x)u_{xy} + (\sin^2 x)u_{yy} + (\cos x)u_y = 0;\quad u(0,y) = y^2,\ u_x(0,y) = y^3.$

3. $y^4\,u_{xx} + 2y^2\,u_{xy} + u_{yy} - \dfrac{2}{y}\,u_y = 0;\quad u(x,1) = \dfrac{x^3}{3},\ u_y(x,1) = 2x.$

4. $4y^2\,u_{xx} + 2(1-y^2)u_{xy} - u_{yy} - \dfrac{4y}{1+y^2}\,u_x + \dfrac{2y}{1+y^2}u_y = 0;\quad u(x,1) = x,\ u_y(x,1) = 0.$

5. $y^2\,u_{xx} - 2y\,u_{xy} + u_{yy} - u_x = 0;\quad u(x,1) = \left(x + \dfrac{1}{2}\right)^2,\ u_y(x,1) = 0.$

6. $u_{xx} - 2(\cos x)u_{xy} - (3 + \sin^2 x)u_{yy} + (\sin x)u_y = 0;\quad u(0,y) = 0,\ u_x(0,y) = y^2.$

7. $y^2\,u_{xx} - 2y\,u_{xy} + u_{yy} + u_x - \dfrac{2}{y}\,u_y = 0;\quad u(x,1) = x^2,\ u_y(x,1) = x.$

8. $u_{xx} - 2(\sin x)u_{xy} - (\cos^2 x)u_{yy} - u_x + (\sin x - \cos x - 1)u_y = 0;\quad u(0,y) = 3y,\ u_x(0,y) = 5.$

9. $u_{xx} + 2(\sin x)u_{xy} + (\sin^2 x)u_{yy} - u_x - (\sin x - \cos x)u_y = 0;\quad u(0,y) = y^2,\ u_x(0,y) = y.$

10. $u_{xx} + 2(\cos x)u_{xy} - (\sin^2 x)u_{yy} - (\sin x)u_y = 0;\quad u(0,y) = y^2,\ u_x(0,y) = 1.$

11. $u_{xx} + 2x^2\,u_{xy} + x^4\,u_{yy} + u_x + (x^2 + 2x)u_y = 0;\quad u(0,y) = y^2,\ u_x(0,y) = y.$

12. $4y^3\,u_{xx} - y\,u_{yy} + 2y^3\,u_x + (1 + y^2)u_y = 0;\quad u(x,1) = x^2,\ u_y(x,1) = 0.$

13. $9y^5\,u_{xx} - y\,u_{yy} + 18y^5\,u_x + (2 - 6y^3)u_y = 0;\quad u(x,1) = 0,\ u_y(x,1) = x.$

14. $u_{xx} - 2(\sin x)u_{xy} - (\cos^2 x)u_{yy} - 2u_x + (2\sin x + 2 - \cos x)u_y = 0;\quad u(0,y) = \dfrac{y^2}{2},\ u_x(0,y) = 1.$

15. $-x\,u_{xx} + 4x^3\,u_{yy} + (1 - 4x^2)u_x + 8x^3\,u_y = 0;\quad u(1,y) = y,\ u_x(1,y) = 3.$

16. $y^2\,u_{xx} - 2y\,u_{xy} + u_{yy} - \dfrac{1}{y}\,u_y = 0;\quad u(x,1) = x,\ u_y(x,1) = x^2.$

17. $u_{xx} - 2(\sin x)u_{xy} - (\cos^2 x)u_{yy} + 2u_x - (2 + \cos x + 2\sin x)u_y = 0;\quad u(0,y) = 2y,\ u_x(0,y) = 1.$

18. $u_{xx} + 2x^2\,u_{xy} + x^4\,u_{yy} + 2x\,u_y = 0;\quad u(0,y) = y,\ u_x(0,y) = y^2.$

19. $u_{xx} + 2x^2\,u_{xy} + x^4\,u_{yy} - u_x + (2x - x^2)u_y = 0;\quad u(0,y) = \sin y,\ u_x(0,y) = y.$

20. $y^2\,u_{xx} + 2y\,u_{xy} + u_{yy} + (1 - y)u_x - u_y = 0;\quad u(x,0) = x^2,\ u_y(x,0) = x.$

21. $-x\,u_{xx} + 9x^5\,u_{yy} + (2 - 6x^3)u_x + 18x^5\,u_y = 0;\quad u(1,y) = 0,\ u_x(1,y) = y.$

22. $y^2\,u_{xx} + 2y\,u_{xy} + u_{yy} + (1 + y)u_x + u_y = 0;\quad u(x,0) = -x,\ u_y(x,0) = \sin x.$

23. $u_{xx} - 2(\sin x)u_{xy} - (\cos^2 x)u_{yy} + u_x + (1 - \cos x - \sin x)u_y = 0;\quad u(0,y) = y,\ u_x(0,y) = 0.$

24. $y^2\,u_{xx} + 2y\,u_{xy} + u_{yy} + u_x = 0;\quad u(x,0) = x^3,\ u_y(x,0) = -x.$

25. $(\sin^2 y)u_{xx} + 2(\cos y)u_{xy} - u_{yy} - (\sin y)u_x = 0;\quad u(x,0) = x^2,\ u_y(x,0) = 1.$

26. $u_{xx} - 2x\,u_{xy} + x^2\,u_{yy} - u_x + (x - 1)u_y = 0;\quad u(0,y) = y,\ u_x(0,y) = y^2.$

27. $(3 + \sin^2 y)u_{xx} - 2(\cos y)u_{xy} - u_{yy} + (\sin y)u_x = 0;\quad u(x,0) = x,\ u_y(x,0) = x^2.$

28. $9y^5\,u_{xx} - y\,u_{yy} + 6y^5\,u_x + (2 + 2y^3)u_y = 0;\quad u(x,1) = 2x,\ u_y(x,1) = 0.$

29. $(\cos^2 y)u_{xx} - 2(\sin y)u_{xy} - u_{yy} + (1 - \cos y + \sin y)u_x + u_y = 0;\quad u(x,0) = x^2,\ u_y(x,0) = 0.$

30. $-x\,u_{xx} + 4x^3\,u_{yy} + (1 + x^2)u_x + 2x^3\,u_y = 0;\quad u(1,y) = y^2,\ u_x(1,y) = 0.$

Задача 3 (*2 балла*)

Решить краевую задачу для уравнения Лапласа в прямоугольнике.

1. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = \dfrac{y-b}{8b}, \ \ u'_x\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = -\dfrac{1}{2a}\sin\dfrac{\pi x}{2a}, \ \ u\big|_{y=b} = \sin\dfrac{5\pi x}{2a}. \end{cases}$$

2. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y(2b-y)}{4b^3}, \ \ u\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = -\dfrac{3}{2a}\cos\dfrac{3\pi x}{2a}, \ \ u'_y\big|_{y=b} = \dfrac{1}{2a}\cos\dfrac{3\pi x}{2a}. \end{cases}$$

3. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = \dfrac{y-b}{8b}, \ \ u\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = -\dfrac{2}{a}\sin\dfrac{2\pi x}{a}, \ \ u\big|_{y=b} = \sin\dfrac{\pi x}{a}. \end{cases}$$

4. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y-b}{2b}, \ \ u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = 1, \ \ u\big|_{y=b} = \cos\dfrac{\pi x}{a}. \end{cases}$$

5. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = \dfrac{y-b}{2b}, \ \ u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = -\sin\dfrac{5\pi x}{2a}, \ \ u\big|_{y=b} = \sin\dfrac{\pi x}{2a}. \end{cases}$$

6. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y(y-2b)}{64b^2}, \ \ u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = -\cos\dfrac{\pi x}{a}, \ \ u'_y\big|_{y=b} = \dfrac{3}{a}\cos\dfrac{3\pi x}{a}. \end{cases}$$

7. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = \dfrac{y(y-2b)}{32b^2}, \ \ u\big|_{x=a} = 0; \\ u\big|_{y=0} = -\sin\dfrac{2\pi x}{a}, \ \ u'_y\big|_{y=b} = \dfrac{1}{a}\sin\dfrac{\pi x}{a}. \end{cases}$$

8. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\cos\dfrac{5\pi y}{2b}, \ \ u'_x\big|_{x=a} = \dfrac{1}{2b}\cos\dfrac{\pi y}{2b}; \\ u'_y\big|_{y=0} = \dfrac{x(x-2a)}{64a^3}, \ \ u\big|_{y=b} = 0. \end{cases}$$

9. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = -\dfrac{1}{b}\cos\dfrac{\pi y}{b}, \ \ u\big|_{x=a} = 1; \\ u'_y\big|_{y=0} = \dfrac{x-a}{16a^2}, \ \ u'_y\big|_{y=b} = 0. \end{cases}$$

10. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\cos\dfrac{\pi y}{2b}, \ \ u\big|_{x=a} = \cos\dfrac{\pi y}{2b}; \\ u'_y\big|_{y=0} = \dfrac{x-a}{2a^2}, \ \ u\big|_{y=b} = 0. \end{cases}$$

11. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\dfrac{1}{b}\sin\dfrac{\pi y}{b}, \ \ u'_x\big|_{x=a} = \dfrac{1}{3b}\sin\dfrac{\pi y}{b}; \\ u\big|_{y=0} = \dfrac{x(2a-x)}{4a^2}, \ \ u\big|_{y=b} = 0. \end{cases}$$

12. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\sin\dfrac{\pi y}{b}, \ \ u'_x\big|_{x=a} = \dfrac{2}{b}\sin\dfrac{2\pi y}{b}; \\ u\big|_{y=0} = \dfrac{x(x-2a)}{32a^2}, \ \ u\big|_{y=b} = 0. \end{cases}$$

13. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = -\dfrac{1}{2b}\sin\dfrac{\pi y}{2b}, \ \ u'_x\big|_{x=a} = \dfrac{3}{2b}\sin\dfrac{3\pi y}{2b}; \\ u\big|_{y=0} = \dfrac{x(2a-x)}{4a^2}, \ \ u'_y\big|_{y=b} = 0. \end{cases}$$

14. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\sin\dfrac{\pi y}{2b}, \ \ u\big|_{x=a} = \sin\dfrac{5\pi y}{2b}; \\ u\big|_{y=0} = \dfrac{x-a}{2a}, \ \ u'_y\big|_{y=b} = 0. \end{cases}$$

15. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = 0, \ \ u'_x\big|_{x=a} = \dfrac{b-y}{16b^2}; \\ u'_y\big|_{y=0} = -\dfrac{3}{2a}\sin\dfrac{3\pi x}{2a}, \ \ u\big|_{y=b} = \sin\dfrac{\pi x}{2a}. \end{cases}$$

16. $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = 0, \ \ u\big|_{x=a} = \dfrac{y(y-2b)}{4b^2}; \\ u'_y\big|_{y=0} = -\dfrac{1}{2a}\cos\dfrac{\pi x}{2a}, \ \ u'_y\big|_{y=b} = \dfrac{3}{2a}\cos\dfrac{3\pi x}{2a}. \end{cases}$$

17. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = 0, \;\; u\big|_{x=a} = \dfrac{b-y}{8b}; \\ u'_y\big|_{y=0} = -\dfrac{4}{a}\sin\dfrac{4\pi x}{a}, \;\; u\big|_{y=b} = \sin\dfrac{\pi x}{a}. \end{cases}$

18. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = 0, \;\; u'_x\big|_{x=a} = \dfrac{b-y}{2b^2}; \\ u\big|_{y=0} = -\cos\dfrac{2\pi x}{a}, \;\; u\big|_{y=b} = 1. \end{cases}$

19. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = 0, \;\; u'_x\big|_{x=a} = \dfrac{b-y}{2b^2}; \\ u\big|_{y=0} = \sin\dfrac{\pi x}{2a}, \;\; u\big|_{y=b} = \sin\dfrac{3\pi x}{2a}. \end{cases}$

20. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = 0, \;\; u'_x\big|_{x=a} = \dfrac{y(2b-y)}{64y_0^3}; \\ u\big|_{y=0} = 1, \;\; u'_y\big|_{y=b} = \dfrac{1}{a}\cos\dfrac{\pi x}{a}. \end{cases}$

21. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = 0, \;\; u\big|_{x=a} = \dfrac{y(2b-y)}{32b^2}; \\ u\big|_{y=0} = -\sin\dfrac{2\pi x}{a}, \;\; u'_y\big|_{y=b} = \dfrac{2}{a}\sin\dfrac{2\pi x}{a}. \end{cases}$

22. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\cos\dfrac{\pi y}{2b}, \;\; u'_x\big|_{x=a} = \dfrac{1}{2b}\cos\dfrac{\pi y}{2b}; \\ u'_y\big|_{y=0} = 0, \;\; u\big|_{y=b} = \dfrac{x(2a-x)}{32a^2}. \end{cases}$

23. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = -\dfrac{2}{b}\cos\dfrac{2\pi y}{b}, \;\; u\big|_{x=a} = 1; \\ u'_y\big|_{y=0} = 0, \;\; u'_y\big|_{y=b} = \dfrac{a-x}{16a^2}. \end{cases}$

24. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\cos\dfrac{3\pi y}{2b}, \;\; u\big|_{x=a} = \cos\dfrac{\pi y}{2b}; \\ u'_y\big|_{y=0} = 0, \;\; u\big|_{y=b} = \dfrac{a-x}{2a}. \end{cases}$

25. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = -\dfrac{3}{b}\sin\dfrac{3\pi y}{b}, \;\; u'_x\big|_{x=a} = \dfrac{1}{b}\sin\dfrac{\pi y}{b}; \\ u\big|_{y=0} = 0, \;\; u\big|_{y=b} = \dfrac{x(x-2a)}{4a^2}. \end{cases}$

26. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y^2}{b^2}, \;\; u\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = \dfrac{3}{2a}\cos\dfrac{3\pi x}{2a}, \;\; u'_y\big|_{y=b} = \dfrac{1}{2a}\cos\dfrac{\pi x}{2a}. \end{cases}$

27. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = \dfrac{b^2-y^2}{8b^2}, \;\; u\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = \dfrac{2}{a}\sin\dfrac{\pi x}{a}, \;\; u\big|_{y=b} = \sin\dfrac{2\pi x}{a}. \end{cases}$

28. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y(b-y)}{2b^2}, \;\; u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = 1, \;\; u\big|_{y=b} = \cos\dfrac{\pi x}{a}. \end{cases}$

29. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = 1, \;\; u\big|_{x=a} = \dfrac{1}{b}\cos\dfrac{\pi y}{b}; \\ u'_y\big|_{y=0} = \dfrac{x^2-a^2}{16a^2}, \;\; u'_y\big|_{y=b} = 0. \end{cases}$

30. $\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y(y-b)}{2b^2}, \;\; u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = \cos\dfrac{2\pi x}{a}, \;\; u\big|_{y=b} = \cos\dfrac{\pi x}{a}. \end{cases}$

Задача 4 (*2 балла*)

Функция f представляет собой ломаную, соединяющую точки A, B, C, D, E, F. Левее точки A и правее точки F она равна нулю. Функция g задана формулой: $g(x) = \operatorname{rect} x$. Найти:

а) преобразование Фурье функции f, используя преобразования Фурье стандартных функций $\operatorname{rect} x$ и $\Lambda(x)$, а также свойства преобразования Фурье;

б) преобразование Фурье свертки $f * g$.

Вар.	***A***	***B***	***C***	***D***	***E***	***F***
1	(0, 0)	(1, −2)	(2, −1)	(3, −1)	(4, −2)	(5, 0)
2	(0, 1)	(1, 3)	(2, 2)	(3, 3)	(4, 1)	(6, 1)
3	(0, 2)	(1, 1)	(2, 1)	(4, 3)	(5, 3)	(6, 2)
4	(0, 0)	(1, 2)	(2, 3)	(3, 3)	(4, 2)	(5, 0)
5	(0, 2)	(2, 4)	(3, 4)	(6, 1)	(7, 1)	(8, 2)
6	(0, 0)	(1, 2)	(2, 2)	(3, 1)	(4, 2)	(5, 0)
7	(0, 0)	(2, −2)	(3, −1)	(4, −2)	(5, −2)	(7, 0)
8	(0, 1)	(3, 4)	(4, 3)	(5, 3)	(7, 1)	(9, 1)
9	(0, 0)	(3, 3)	(4, 2)	(5, 2)	(6, 3)	(9, 0)
10	(0, 1)	(3, 4)	(4, 3)	(5, 3)	(8, 0)	(9, 1)
11	(0, 3)	(1, 1)	(2, 0)	(3, 0)	(4, 1)	(5, 3)
12	(0, 2)	(1, 0)	(2, 1)	(3, 0)	(4, 2)	(7, 2)
13	(0, 1)	(1, 1)	(3, 3)	(4, 3)	(5, 4)	(8, 1)
14	(1, 1)	(3, 3)	(4, 2)	(5, 3)	(6, 3)	(8, 1)
15	(0, −2)	(1, −3)	(2, −3)	(5, 0)	(7, −2)	(9, −2)
16	(0, 1)	(1, 1)	(2, 2)	(5, 2)	(6, 1)	(7, 1)
17	(0, 0)	(1, 2)	(2, 3)	(3, 2)	(4, 2)	(5, 0)
18	(0, 1)	(2, 3)	(3, 3)	(4, 4)	(8, 0)	(9, 1)
19	(0, −1)	(1, −1)	(2, 0)	(5, −3)	(7, −3)	(9, −1)
20	(0, −1)	(2, 1)	(3, 0)	(5, 2)	(8, −1)	(9, −1)
21	(0, 3)	(2, 3)	(4, 1)	(5, 2)	(7, 0)	(10, 3)
22	(0, 0)	(2, 2)	(4, 2)	(5, 1)	(6, 1)	(7, 0)
23	(0, 0)	(1, −1)	(2, −1)	(3, −2)	(5, −2)	(7, 0)
24	(−2, 3)	(0, 5)	(4, 1)	(5, 2)	(7, 0)	(10, 3)
25	(2, 3)	(4, 1)	(5, 2)	(7, 0)	(12, 5)	(14, 3)
26	(1, 1)	(3, 3)	(4, 3)	(5, 2)	(6, 3)	(8, 1)
27	(0, 1)	(1, 0)	(4, 3)	(5, 3)	(6, 4)	(9, 1)
28	(0, 2)	(1, 1)	(4, 1)	(6, 3)	(7, 3)	(8, 2)
29	(0, 1)	(1, 1)	(2, 3)	(3, 2)	(4, 1)	(5, 1)
30	(0, 1)	(1, 1)	(2, 3)	(3, 3)	(5, 1)	(6, 1)

Примечание 1.

$$\mathcal{F}[\operatorname{rect}](\omega) = \operatorname{sinc}\frac{\omega}{2}; \quad \mathcal{F}[\Lambda](\omega) = \operatorname{sinc}^2\frac{\omega}{2}.$$

Примечание 2.

$$\operatorname{rect} x = \eta\Big(\frac{1}{2} - |x|\Big); \quad \Lambda(x) = (1 - |x|)\,\eta(1 - |x|)); \quad \operatorname{sinc} x = \frac{\sin x}{x}.$$