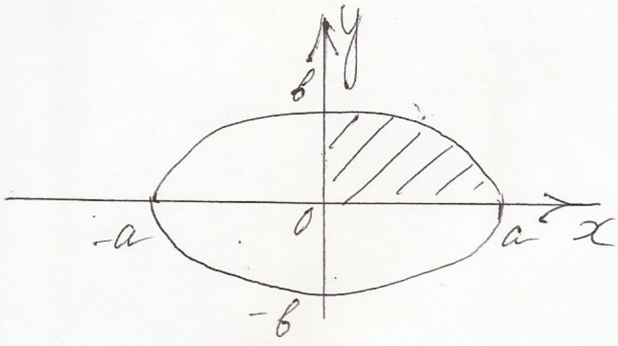
3) Найти площадь области плоскости $xoy$ ограниченную эллипсом $\begin{cases} x = a\cos t, \\ y = b\sin t. \end{cases}$




$$S = \int_{t_1}^{t_2} y(t)\, x'_t \, dt$$

$$\begin{cases} x(t) = a\cos t, \\ y(t) = b\sin t \end{cases}$$

$$dx = -a\sin t\, dt$$

$\frac{1}{4}S: \; 0 \le x \le 0$

$(0;b) \longrightarrow (a;0)$

$\quad t_1 \qquad\qquad t_2$

$t_1: \begin{cases} 0 = a\cos t_1, \\ b = b\sin t_1 \end{cases} \Rightarrow \begin{cases} \cos t_1 = 0 \\ \sin t_1 = 1 \end{cases} \; t_1 = \frac{\pi}{2}.$

$t_2: \begin{cases} a = a\cos t_2, \\ 0 = b\sin t_2 \end{cases} \Rightarrow \begin{cases} \cos t_2 = 1 \\ \sin t_2 = 0 \end{cases} \Rightarrow t_2 = 0$

$\Rightarrow t_1$ от $\frac{\pi}{2}$ до $0$

$$S = 4\int_{\frac{\pi}{2}}^{0} b\sin t \cdot (-a\sin t)\, dt = -4ab\int_{\frac{\pi}{2}}^{0} \sin^2 t\, dt =$$

$$= 4ab\int_0^{\pi/2} \frac{1-\cos 2t}{2}\, dt = \frac{4ab}{2}\left(\int_0^{\pi/2} dt - \frac{1}{2}\int_0^{\pi/2} \cos 2t\, d(2t)\right) =$$

$$= 2ab\left(t\Big|_0^{\pi/2} - \frac{1}{2}\sin 2t\Big|_0^{\pi/2}\right) = 2ab\left(\frac{\pi}{2} - 0 - \frac{1}{2}\left(\sin\frac{2\pi}{2} - \sin 0\right)\right) =$$

$$= 2ab\left(\frac{\pi}{2} - 0 - \frac{1}{2}\cdot 0\right) = 2ab\cdot\frac{\pi}{2} = \pi ab.$$

Ответ: $S = \pi ab$.