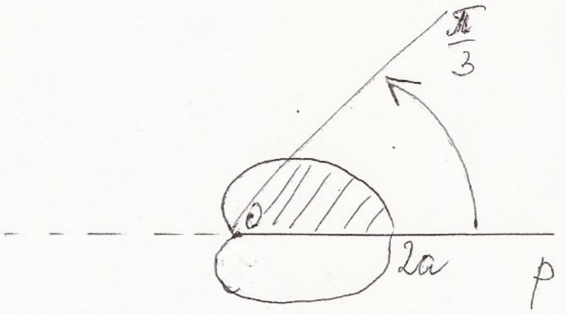
Вариант 0. (РК 1 Инт и ДУ)

① Найти площадь фигуры, ограниченной кардиоидой $\rho = a(1+\cos\varphi)$ и лучами $\varphi = 0$, $\varphi = \frac{\pi}{3}$. Сделать чертеж.



$$S = \frac{1}{2}\int_{\alpha}^{\beta} \rho^2(\varphi)\,d\varphi$$

$$S = \frac{1}{2}\int_0^{\frac{\pi}{3}} \left(a(1+\cos\varphi)\right)^2 d\varphi = \frac{a^2}{2}\int_0^{\frac{\pi}{3}} \left(1^2 + 2\cos\varphi + \cos^2\varphi\right) d\varphi =$$

$$= \frac{a^2}{2}\left(\varphi\Big|_0^{\frac{\pi}{3}} + 2\sin\varphi\Big|_0^{\frac{\pi}{3}} + \frac{1}{2}\int_0^{\pi/3}(1+\cos 2\varphi)\right) d\varphi =$$

$$= \frac{a^2}{2}\left(\left(\frac{\pi}{3}-0\right) + 2\left(\sin\frac{\pi}{3} - \sin 0\right) + \frac{1}{2}\varphi\Big|_0^{\frac{\pi}{3}} + \frac{1}{4}\sin 2\varphi\Big|_0^{\pi/3}\right) =$$

$$= \frac{a^2}{2}\left(\frac{\pi}{3}^{(2} + 2\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\,\frac{\pi}{3} + \frac{1}{4}\left(\sin\frac{2\pi}{3} - \sin 0\right)\right) =$$

$$= \frac{a^2}{2}\left(\frac{\pi}{2} + \sqrt{3} + \frac{1}{4}\sin\left(\pi - \frac{\pi}{3}\right)\right) = \frac{a^2}{2}\left(\frac{\pi}{2} + \sqrt{3} + \frac{1}{4}\sin\frac{\pi}{3}\right) =$$

$$= \frac{a^2}{2}\left(\frac{\pi}{2} + \sqrt{3}^{(8} + \frac{1}{4}\cdot\frac{\sqrt{3}}{2}\right) = \frac{a^2}{2}\left(\frac{\pi}{2} + \frac{9\sqrt{3}}{8}\right).$$

Ответ: $\frac{a^2}{2}\left(\frac{\pi}{2} + \frac{9\sqrt{3}}{8}\right)$.