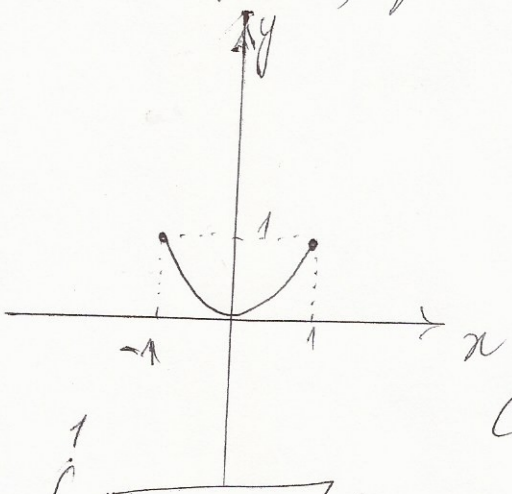
$$= -2\pi\left(\frac{4^2\cdot 4^2}{8} - \frac{2\cdot 4\cdot 4^2}{3}\right) = -2\pi\cdot 4^3\left(\frac{1}{2}-\frac{2}{3}\right) =$$

$$= 2\pi\cdot 4^3\left(\frac{2}{3}-\frac{1}{2}\right)^{(2} = 2\pi\cdot 4^3\cdot\frac{1}{6} = \frac{64\pi}{3}.$$

Ответ: $\frac{64\pi}{3}$.

③ Найти длину дуги кривой $y = x^2$ от точки $(-1;1)$ до точки $(1;1)$. Сделать чертеж.



$$L = \int_a^b \sqrt{1+(f'_x)^2}\,dx$$

$$y'_x = 2x$$

С учетом симметрии дуги:

$$L = 2\cdot\int_0^1 \sqrt{1+4x^2}\,dx \;\div$$

$$\int\sqrt{1+4x^2}\,dx = \left\{\begin{array}{l|l} u=\sqrt{1+4x^2} & du = \frac{8x}{2\sqrt{1+4x^2}}dx \\ dv = dx & v = x \end{array}\right\} =$$

$$= x\sqrt{1+4x^2} - \int\frac{4x^2}{\sqrt{1+4x^2}}dx = x\sqrt{1+4x^2} - \int\frac{(1+4x^2)-1}{\sqrt{1+4x^2}}dx =$$

$$= x\sqrt{1+4x^2} - \int\sqrt{1+4x^2}\,dx + \int\frac{dx}{\sqrt{4(x^2+\frac{1}{4})}} =$$

$$= x\sqrt{1+4x^2} - \int\sqrt{1+4x^2}dx + \frac{1}{2}\ln\left|x+\sqrt{x^2+\frac{1}{4}}\right|;$$

$$2\int\sqrt{1+4x^2}\,dx = x\sqrt{1+4x^2} + \frac{1}{2}\ln\left|x+\sqrt{x^2+\frac{1}{4}}\right|;$$

$$\int\sqrt{1+4x^2}\,dx = \frac{x}{2}\sqrt{1+4x^2} + \frac{1}{4}\ln\left|x+\sqrt{x^2+\frac{1}{4}}\right|;$$