

Привести к каноническому виду

$$2x^2 + 2y^2 + 2z^2 - 2xz + \sqrt{2}x - 4y + \sqrt{2}z - 9 = 0$$

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$$

$$\det_{A-\lambda E} \begin{vmatrix} 2-\lambda & 0 & -1 \\ 0 & 2-\lambda & 0 \\ -1 & 0 & 2-\lambda \end{vmatrix} = (2-\lambda)(2-\lambda)^2 - 1(0 + (2-\lambda)) =$$

$$= (2-\lambda)^3 - (2-\lambda) = (2-\lambda)[(2-\lambda)^2 - 1] = (2-\lambda)(2-\lambda-1)(2-\lambda+1) =$$

$$= (2-\lambda)(1-\lambda)(3-\lambda)$$

$$\lambda_1 = 1 \Rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} y=0 \\ x=z \end{matrix} \quad \left((1; 0; 1) \cdot \frac{1}{\sqrt{2}} \right) e_1$$

$$\lambda_2 = 2 \Rightarrow \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{matrix} z=0 \\ y \text{ — любое} \\ y=0 \end{matrix} \quad (0; 1; 0) = \bar{e}_2$$

$$\Downarrow \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{matrix} y=0 \\ x=-z \end{matrix} \quad \left((-1; 0; 1) \cdot \frac{1}{\sqrt{2}} \right) \bar{e}_3$$

$$T = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{cases} x = \frac{1}{\sqrt{2}}x' - \frac{1}{\sqrt{2}}z' \\ y = y' \\ z = \frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}z' \end{cases}$$