$$L = \int_0^{\frac{\pi}{2}} \sqrt{25\cos^8 t \sin^2 t + 25 \sin^8 t \cos^2 t}\, dt =$$

$$= 5 \int_0^{\frac{\pi}{2}} \sin t \cos t \sqrt{\sin^6 t + \cos^6 t}\, dt = \left\{ \begin{array}{l} \sin^6 t + \cos^6 t = \\ = (\sin^2 t)^3 + (\cos^2 t)^3 = \end{array} \right.$$

$$= \underbrace{(\sin^2 t + \cos^2 t)}_{1} (\sin^4 t - \sin^2 t \cos^2 t + \cos^4 t) =$$

$$\# \ \sin^2 t (\sin^2 t - \cos^2 t) + \cos^4 t = -\sin^2 t \cos 2t + \cos^4 t$$

$$= \left((\sin^2 t)^2 + (\cos^2 t)^2 + 2\cos^2 t \sin^2 t\right) - 2\cos^2 t \sin^2 t - \sin^2 t \cos^2 t =$$

$$= (\sin^2 t + \cos^2 t)^2 - 3\cos^2 t \sin^2 t = 1 - \frac{3}{4} \sin^2 2t =$$

$$= 1 - \frac{3}{4}(1 - \cos^2 2t) = 1 - \frac{3}{4} + \frac{3}{4}\cos^2 2t = \frac{1}{4} + \frac{3}{4}\cos^2 2t \ \Big\} =$$

$$= 5 \int_0^{\frac{\pi}{2}} \frac{1}{2} \sin 2t \sqrt{\frac{1}{4} + \frac{3}{4}\cos^2 2t}\, dt = -\frac{5}{2 \cdot 2} \int_0^{\frac{\pi}{2}} \sqrt{\frac{1}{4} + \frac{3}{4}\cos^2 2t}\, d\cos 2t =$$

$$= -\frac{5}{4} \int_0^{\frac{\pi}{2}} \frac{1}{2} \sqrt{1 + 3\cos^2 2t}\, d\cos 2t = -\frac{5}{8} \int_0^{\frac{\pi}{2}} \sqrt{1 + 3\cos^2 2t}\, d\cos 2t =$$

$$= \left\{ \int \sqrt{1 + 3u^2}\, du \ \text{по частям} \right\} = -\frac{5}{8\sqrt{3}} \left[\frac{\sqrt{3}}{2} \cos 2t \sqrt{1 + 3\cos^2 2t} + \frac{1}{2} \ln\left(\sqrt{3}\cos 2t + \sqrt{1 + 3\cos^2 2t}\right) \right] \Bigg|_0^{\frac{\pi}{2}} = \frac{5}{8}\left[2 - \frac{\ln(2 - \sqrt{3})}{\sqrt{3}}\right].$$

Ответ: $\frac{5}{8}\left[2 - \frac{\ln(2-\sqrt{3})}{\sqrt{3}}\right]$