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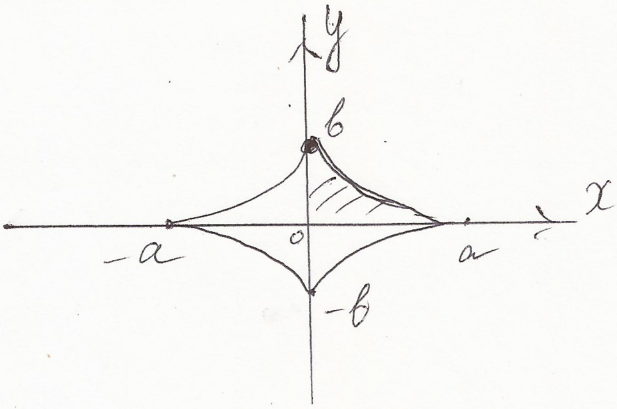
2) Найти площадь, содержащуюся внутри астроиды $\begin{cases} x = a\cos^3 t, \\ y = b\sin^3 t. \end{cases}$



$$S = \int_{t_1}^{t_2} y(t)\cdot \varphi'_t \, dt$$

$x(t) = a\cos^3 t$

$dx = (a\cos^3 t)' dt$

$dx = -3a\cos^2 t \sin t \, dt$

$\frac{1}{4} S: \quad \underset{t_1}{(0;b)} \to \underset{t_2}{(a;0)}$

$t_1: \begin{cases} 0 = a\cos^3 t_1, \\ b = b\sin^3 t_1. \end{cases} \Rightarrow \left.\begin{matrix} \cos t_1 = 0 \\ \sin t_1 = 1 \end{matrix}\right\} \Rightarrow t_1 = \frac{\pi}{2}$

$t_2: \begin{cases} a = a\cos^3 t_2 \\ 0 = b\sin^3 t_2 \end{cases} \Rightarrow \begin{cases} \cos t_2 = 1 \\ \sin t_2 = 0 \end{cases} \Rightarrow t_2 = 0.$

$$S = 4\int_{\frac{\pi}{2}}^{0} b\sin^3 t \cdot (-3a\cos^2 t \sin t)\,dt =$$

$$= -4\cdot 3ab \int_{\frac{\pi}{2}}^{0} \sin^4 t \cos^2 t \, dt = \left\{ \int_c^d f(t)\,dt = -\int_d^c f(t)\,dt \right\} =$$

$$= 12ab \int_0^{\pi/2} \left(\frac{1-\cos 2t}{2}\right)^2 \cdot \left(\frac{1+\cos 2t}{2}\right) dt = \frac{3\pi ab}{8}.$$