

Задача 1

а. Решить краевую задачу для уравнения Лапласа в прямоугольнике. (2 балла)

1.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = \frac{y-b}{8b}, & u'|_{x=a} = 0; \\ u'|_{y=0} = -\frac{1}{2a} \sin \frac{\pi x}{2a}, & u|_{y=b} = \sin \frac{5\pi x}{2a}. \end{cases}$$
2.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'|_{x=0} = \frac{y(2b-y)}{4b^3}, & u|_{x=a} = 0; \\ u'|_{y=0} = -\frac{3}{2a} \cos \frac{3\pi x}{2a}, & u'|_{y=b} = \frac{1}{2a} \cos \frac{3\pi x}{2a}. \end{cases}$$
3.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = \frac{y-b}{8b}, & u|_{x=a} = 0; \\ u'|_{y=0} = -\frac{2}{a} \sin \frac{2\pi x}{a}, & u|_{y=b} = \sin \frac{\pi x}{a}. \end{cases}$$
4.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'|_{x=0} = \frac{y-b}{2b}, & u'|_{x=a} = 0; \\ u|_{y=0} = 1, & u|_{y=b} = \cos \frac{\pi x}{a}. \end{cases}$$
5.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = \frac{y-b}{2b}, & u'|_{x=a} = 0; \\ u|_{y=0} = -\sin \frac{5\pi x}{2a}, & u|_{y=b} = \sin \frac{\pi x}{2a}. \end{cases}$$
6.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'|_{x=0} = \frac{y(y-2b)}{64b^2}, & u'|_{x=a} = 0; \\ u|_{y=0} = -\cos \frac{\pi x}{a}, & u'|_{y=b} = \frac{3}{a} \cos \frac{3\pi x}{a}. \end{cases}$$
7.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = \frac{y(y-2b)}{32b^2}, & u|_{x=a} = 0; \\ u|_{y=0} = -\sin \frac{2\pi x}{a}, & u'|_{y=b} = \frac{1}{a} \sin \frac{\pi x}{a}. \end{cases}$$
8.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = -\cos \frac{5\pi y}{2b}, & u'|_{x=a} = \frac{1}{2b} \cos \frac{\pi y}{2b}; \\ u'|_{y=0} = \frac{x(x-2a)}{64a^3}, & u|_{y=b} = 0. \end{cases}$$
9.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'|_{x=0} = -\frac{1}{b} \cos \frac{\pi y}{b}, & u|_{x=a} = 1; \\ u'|_{y=0} = \frac{x-a}{16a^2}, & u'|_{y=b} = 0. \end{cases}$$
10.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = -\cos \frac{\pi y}{2b}, & u|_{x=a} = \cos \frac{\pi y}{2b}; \\ u'|_{y=0} = \frac{x-a}{2a^2}, & u|_{y=b} = 0. \end{cases}$$
11.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'|_{x=a} = -\frac{1}{b} \sin \frac{\pi y}{b}, & u'|_{x=a} = \frac{1}{3b} \sin \frac{\pi y}{b}; \\ u|_{y=0} = \frac{x(2a-x)}{4a^2}, & u|_{y=b} = 0. \end{cases}$$
12.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = -\sin \frac{\pi y}{b}, & u'|_{x=a} = \frac{2}{b} \sin \frac{2\pi y}{b}; \\ u|_{y=0} = \frac{x(x-2a)}{32a^2}, & u|_{y=b} = 0. \end{cases}$$
13.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'|_{x=0} = -\frac{1}{2b} \sin \frac{\pi y}{2b}, & u'|_{x=a} = \frac{3}{2b} \sin \frac{3\pi y}{2b}; \\ u|_{y=0} = \frac{x(2a-x)}{4a^2}, & u'|_{y=b} = 0. \end{cases}$$
14.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = -\sin \frac{\pi y}{2b}, & u|_{x=a} = \sin \frac{5\pi y}{2b}; \\ u|_{y=0} = \frac{x-a}{2a}, & u'|_{y=b} = 0. \end{cases}$$
15.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = 0, & u'|_{x=a} = \frac{b-y}{16y_0^2}; \\ u'|_{y=0} = -\frac{3}{2a} \sin \frac{3\pi x}{2a}, & u|_{y=b} = \sin \frac{\pi x}{2a}. \end{cases}$$
16.
$$\begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'|_{x=0} = 0, & u|_{x=a} = \frac{y(y-2b)}{4y_0^2}; \\ u'|_{y=0} = -\frac{1}{2a} \cos \frac{\pi x}{2a}, & u'|_{y=b} = \frac{3}{2a} \cos \frac{3\pi x}{2a}. \end{cases}$$

$$17. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = 0, & u|_{x=a} = \frac{b-y}{8b}; \\ u'_y|_{y=0} = -\frac{4}{a} \sin \frac{4\pi x}{a}, & u|_{y=b} = \sin \frac{\pi x}{a}. \end{cases}$$

$$18. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'_x|_{x=0} = 0, & u'_x|_{x=a} = \frac{b-y}{2b^2}; \\ u|_{y=0} = -\cos \frac{2\pi x}{a}, & u|_{y=b} = 1. \end{cases}$$

$$19. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = 0, & u'_x|_{x=a} = \frac{b-y}{2b^2}; \\ u|_{y=0} = \sin \frac{\pi x}{2a}, & u|_{y=b} = \sin \frac{3\pi x}{2a}. \end{cases}$$

$$20. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'_x|_{x=0} = 0, & u'_x|_{x=a} = \frac{y(2b-y)}{64y_0^3}; \\ u|_{y=0} = 1, & u'_y|_{y=b} = \frac{1}{a} \cos \frac{\pi x}{a}. \end{cases}$$

$$21. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = 0, & u|_{x=a} = \frac{y(2b-y)}{32b^2}; \\ u|_{y=0} = -\sin \frac{2\pi x}{a}, & u'_y|_{y=b} = \frac{2}{a} \sin \frac{2\pi x}{a}. \end{cases}$$

$$22. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = -\cos \frac{\pi y}{2b}, & u'_x|_{x=a} = \frac{1}{2b} \cos \frac{\pi y}{2b}; \\ u'_y|_{y=0} = 0, & u|_{y=b} = \frac{x(2a-x)}{32a^2}. \end{cases}$$

$$23. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'_x|_{x=0} = -\frac{2}{b} \cos \frac{2\pi y}{b}, & u|_{x=a} = 1; \\ u'_y|_{y=0} = 0, & u'_y|_{y=b} = \frac{a-x}{16a^2}. \end{cases}$$

$$24. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u|_{x=0} = -\cos \frac{3\pi y}{2b}, & u|_{x=a} = \cos \frac{\pi y}{2b}; \\ u'_y|_{y=0} = 0, & u|_{y=b} = \frac{a-x}{2a}. \end{cases}$$

$$25. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'_x|_{x=0} = -\frac{3}{b} \sin \frac{3\pi y}{b}, & u'_x|_{x=a} = \frac{1}{b} \sin \frac{\pi y}{b}; \\ u|_{y=0} = 0, & u|_{y=b} = \frac{x(x-2a)}{4a^2}. \end{cases}$$

$$26. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'_x|_{x=0} = \frac{y^2}{b^2}, & u|_{x=a} = 0; \\ u'_y|_{y=0} = \frac{3}{2a} \cos \frac{3\pi x}{2a}, & u'_y|_{y=b} = \frac{1}{2a} \cos \frac{\pi x}{2a}. \end{cases}$$

$$27. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'_x|_{x=0} = \frac{b^2-y^2}{8b^2}, & u|_{x=a} = 0; \\ u'_y|_{y=0} = \frac{2}{a} \sin \frac{\pi x}{a}, & u|_{y=b} = \sin \frac{2\pi x}{a}. \end{cases}$$

$$28. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'_x|_{x=0} = \frac{y(b-y)}{2b^2}, & u'_x|_{x=a} = 0; \\ u|_{y=0} = 1, & u|_{y=b} = \cos \frac{\pi x}{a}. \end{cases}$$

$$29. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'_x|_{x=0} = 1, & u|_{x=a} = \frac{1}{b} \cos \frac{\pi y}{b}; \\ u'_y|_{y=0} = \frac{x^2-a^2}{16a^2}, & u'_y|_{y=b} = 0. \end{cases}$$

$$30. \begin{cases} \Delta u = 0, & 0 < x < a, & 0 < y < b; \\ u'_x|_{x=0} = \frac{y(y-b)}{2b^2}, & u'_x|_{x=a} = 0; \\ u|_{y=0} = \cos \frac{2\pi x}{a}, & u|_{y=b} = \cos \frac{\pi x}{a}. \end{cases}$$

б. Решить краевую задачу для уравнения Лапласа в круге. (2 балла)

31.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = 2 \cos^3 \varphi - \sin^3 \varphi + \sin \varphi. \end{cases}$$
32.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=1} = 3 \cos^3 \varphi + \sin^3 \varphi - \sin \varphi. \end{cases}$$
33.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 4, & 0 \leq \varphi < 2\pi; \\ u|_{r=4} = \cos^3 \varphi + 4 \sin^3 \varphi + \sin^2 \varphi. \end{cases}$$
34.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u|_{r=3} = 4 \sin^3 \varphi - \sin^2 \varphi + \sin \varphi. \end{cases}$$
35.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = 2 \cos^3 \varphi + \sin \varphi. \end{cases}$$
36.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u|_{r=1} = \cos^3 \varphi - \cos^2 \varphi + \sin \varphi. \end{cases}$$
37.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=3} = 3 \sin^3 \varphi - \cos^3 \varphi + \sin \varphi. \end{cases}$$
38.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 4, & 0 \leq \varphi < 2\pi; \\ u|_{r=4} = 2 \cos^3 \varphi + 4 \sin^3 \varphi - \sin^2 \varphi. \end{cases}$$
39.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=1} = -4 \cos^3 \varphi - \sin^3 \varphi + 2 \sin \varphi. \end{cases}$$
40.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = 4 \cos^3 \varphi - 2 \sin^3 \varphi - 3 \cos \varphi + 2 \sin \varphi. \end{cases}$$
41.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=3} = \sin^3 \varphi - \cos^3 \varphi + 3 \sin \varphi. \end{cases}$$
42.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 6, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=6} = 2 \cos^3 \varphi - 3 \cos^3 \varphi + \cos \varphi. \end{cases}$$
43.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 4, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=4} = 2 \cos^3 \varphi + 2 \sin^3 \varphi + 2 \sin \varphi. \end{cases}$$
44.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u|_{r=1} = -\sin^3 \varphi - 2 \sin^2 \varphi + 3 \sin \varphi. \end{cases}$$
45.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = 4 \cos^3 \varphi - 2 \sin^3 \varphi + \cos^2 \varphi. \end{cases}$$
46.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=3} = 2 \cos^3 \varphi - 4 \sin^3 \varphi + \cos \varphi. \end{cases}$$
47.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 4, & 0 \leq \varphi < 2\pi; \\ u|_{r=4} = 2 \cos^2 \varphi - \sin^3 \varphi + \cos \varphi. \end{cases}$$
48.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 5, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=5} = 2 \sin^3 \varphi - \sin \varphi + 2 \cos \varphi. \end{cases}$$
49.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = 2 \cos^3 \varphi + 4 \sin^3 \varphi + 2 \sin \varphi. \end{cases}$$
50.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=1} = 6 \sin^3 \varphi - \cos^3 \varphi + 4 \sin \varphi. \end{cases}$$
51.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u|_{r=3} = \cos^3 \varphi + \sin^3 \varphi + \sin^2 \varphi. \end{cases}$$
52.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 4, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=4} = 4 \sin^3 \varphi - \sin \varphi - \cos \varphi. \end{cases}$$
53.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 5, & 0 \leq \varphi < 2\pi; \\ u|_{r=5} = -\cos^3 \varphi - \sin^3 \varphi + \cos \varphi. \end{cases}$$
54.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = 4 \cos^3 \varphi + 2 \sin^3 \varphi + \sin \varphi - 3 \cos \varphi. \end{cases}$$
55.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=1} = 2 \cos^3 \varphi - \sin^3 \varphi + 3 \sin \varphi. \end{cases}$$
56.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u|_{r=1} = \sin^3 \varphi + 2 \sin^2 \varphi - 3 \sin \varphi - \cos \varphi. \end{cases}$$
57.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = \cos^3 \varphi + \sin^3 \varphi + 3 \sin^2 \varphi. \end{cases}$$
58.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 5, & 0 \leq \varphi < 2\pi; \\ u|_{r=5} = 4 \sin^3 \varphi + \sin^2 \varphi - \sin \varphi. \end{cases}$$
59.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u|_{r=3} = 4 \cos^3 \varphi + 2 \sin^3 \varphi + 2 \cos \varphi. \end{cases}$$
60.
$$\begin{cases} \Delta u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u|_{r=1} = 4 \cos^3 \varphi - 4 \sin^3 \varphi + \cos^2 \varphi. \end{cases}$$

Задача 2

Решить краевую задачу для уравнения Гельмгольца в круге. (4 балла)

1.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = 2 \cos^3 \varphi - 3 \sin \varphi. \end{cases}$$
2.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=1} = \cos^3 \varphi + \sin \varphi. \end{cases}$$
3.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = 3 \cos^3 \varphi - \sin \varphi. \end{cases}$$
4.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u|_{r=3} = \cos^2 \varphi - 3 \sin \varphi. \end{cases}$$
5.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = \cos^3 \varphi - 2 \sin^3 \varphi. \end{cases}$$
6.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = \sin^3 \varphi + 5 \cos \varphi. \end{cases}$$
7.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u|_{r=1} = \cos^2 \varphi - 3 \sin \varphi. \end{cases}$$
8.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = \cos^3 \varphi + \sin \varphi. \end{cases}$$
9.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u|_{r=3} = \cos^3 \varphi + \sin^2 \varphi - \cos \varphi. \end{cases}$$
10.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=1} = \sin^3 \varphi + 3 \cos \varphi. \end{cases}$$
11.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u|_{r=1} = \cos^2 \varphi - 2 \sin \varphi. \end{cases}$$
12.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = \cos^2 \varphi + 5 \sin \varphi. \end{cases}$$
13.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = \cos^3 \varphi - \sin \varphi. \end{cases}$$
14.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u|_{r=1} = \cos^2 \varphi - 6 \sin^3 \varphi. \end{cases}$$
15.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = \cos^3 \varphi - \sin^2 \varphi. \end{cases}$$
16.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u|_{r=3} = \cos^2 \varphi - 3 \sin \varphi. \end{cases}$$
17.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = 2 \cos^3 \varphi - \sin \varphi. \end{cases}$$
18.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = \cos^2 \varphi + 5 \sin \varphi. \end{cases}$$
19.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = 3 \cos^3 \varphi + 5 \cos \varphi. \end{cases}$$
20.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = \cos^2 \varphi + 4 \sin \varphi. \end{cases}$$
21.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=1} = \cos^3 \varphi + \sin^3 \varphi. \end{cases}$$
22.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = \cos^3 \varphi + 12 \sin^3 \varphi. \end{cases}$$
23.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=3} = \cos^3 \varphi - \sin^3 \varphi. \end{cases}$$
24.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = \cos^2 \varphi - 3 \sin^2 \varphi. \end{cases}$$
25.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = \cos^2 \varphi + 5 \sin \varphi. \end{cases}$$
26.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=2} = \cos^3 \varphi - 2 \sin \varphi. \end{cases}$$
27.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 1, & 0 \leq \varphi < 2\pi; \\ u'_r|_{r=1} = \cos^2 \varphi + 2 \sin \varphi. \end{cases}$$
28.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 3, & 0 \leq \varphi < 2\pi; \\ u|_{r=3} = \sin^2 \varphi - 3 \sin \varphi. \end{cases}$$
29.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 2, & 0 \leq \varphi < 2\pi; \\ u|_{r=2} = \cos^3 \varphi + 2 \sin \varphi. \end{cases}$$
30.
$$\begin{cases} \Delta u + u = 0, & 0 \leq r < 4, & 0 \leq \varphi < 2\pi; \\ u|_{r=4} = \sin^2 \varphi + 6 \sin^3 \varphi. \end{cases}$$