Вычисление определённого интеграла с помощью неопределённого.

Формула Ньютона-Лейбница:

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a).$$

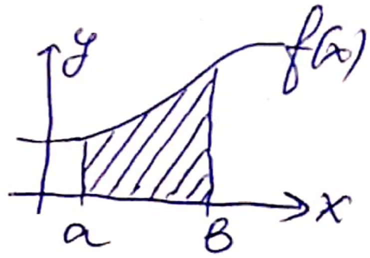
№ 1521.

$$\int_1^2 (x^2-2x+3)\,dx = \left(\frac{x^3}{3} - \cancel{2}\cdot\frac{x^2}{\cancel{2}} + 3x\right)\Bigg|_1^2 =$$

$$= \left(\frac{8}{3} - 4 + 6\right) - \left(\frac{1}{3} - 1 + 3\right) = \frac{7}{3}$$

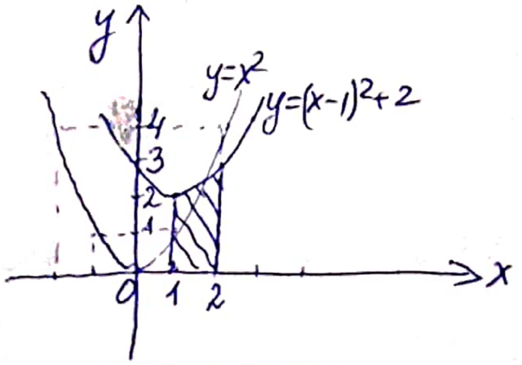
Геом. смысл для $f(x) \geq 0$:

$$\int_a^b f(x)\,dx = S_{\text{криволин. трапеции}}$$



У нас:

$$f(x) = x^2-2x+3 = x^2-2x+1-1+3 = (x-1)^2+2$$



След., $S_{\substack{\text{криволин.}\\ \text{трапеции}}} = \frac{7}{3}$

ДЗ I. № 1522.

№1538

$$\int_e^{e^2} \frac{dx}{x\ln x} = \int_e^{e^2} \frac{d(\ln x)}{\ln x} = \ln|\ln x|\Big|_e^{e^2} = \ln|\ln e^2| - \ln|\ln e| =$$

$$= \ln 2 - \ln 1 = \ln 2$$

Д/З II №1539

№1534.

$$\int_2^{3,5} \frac{dx}{\sqrt{5+4x-x^2}} =$$

$$= \left[\begin{aligned} &5+4x-x^2 = -(x^2-4x-5) = -(x^2-4x+4-4-5) = \\ &= -((x-2)^2-9) = 9-(x-2)^2 \end{aligned} \right] =$$

$$= \int_2^{3,5} \frac{d(x-2)}{\sqrt{3^2-(x-2)^2}} = \arcsin\frac{x-2}{3}\Big|_2^{3,5} =$$

$$= \arcsin\frac{3,5-2}{3} - \arcsin\frac{2-2}{3} =$$

$$= \arcsin\frac{\frac{3}{2}}{3} - \arcsin\frac{0}{3} =$$

$$= \arcsin\frac{1}{2} - \arcsin 0 = \frac{\pi}{6}$$

Д/З III №1529, 1527.

№1536.

3

$$\int_0^{\frac{\pi}{4}} \cos^2\alpha\, d\alpha = \int_0^{\frac{\pi}{4}} \frac{1+\cos 2\alpha}{2}\, d\alpha = \frac{1}{2}\left(\int_0^{\frac{\pi}{4}} d\alpha + \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos 2\alpha\, d(2\alpha)\right) =$$

$$= \frac{1}{2}\left(\alpha\Big|_0^{\frac{\pi}{4}} + \frac{1}{2}\sin 2\alpha\Big|_0^{\frac{\pi}{4}}\right) = \frac{1}{2}\left(\left(\frac{\pi}{4} - 0\right) + \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin 0\right)\right) =$$

$$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$$

Д/З IV. №1537, 1541

Можно оформить вычисление интеграла по-другому:

$$\int_0^{\frac{\pi}{4}} \cos^2\alpha\, d\alpha = I$$

Найдём неопределённый интеграл:

$$\int \cos^2\alpha\, d\alpha = \int \frac{1+\cos 2\alpha}{2}\, d\alpha = \frac{1}{2}\left(\int d\alpha + \frac{1}{2}\int \cos 2\alpha\, d(2\alpha)\right) =$$

$$= \underbrace{\frac{1}{2}\left(\alpha + \frac{1}{2}\sin 2\alpha\right)}_{F(x)} + C$$

Подставим в определённой интеграл:

$$I = \frac{1}{2}\left(\alpha + \frac{1}{2}\sin 2\alpha\right)\Big|_0^{\frac{\pi}{4}} = \frac{1}{2}\left(\left(\frac{\pi}{4} + \frac{1}{2}\sin\frac{\pi}{2}\right) - \left(0 + \frac{1}{2}\sin 0\right)\right) =$$

$$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}.$$

Замена переменных в определённом интеграле

$$\int_a^b f(u(x))\cdot u'(x)dx = \int_a^b f(u(x))du(x) = F(u(x))\Big|_a^b$$

С помощью замены перем.:

то же .... $=$

$$= \begin{bmatrix} t = u(x) \\ x = a \Rightarrow t = \alpha \\ x = b \Rightarrow t = \beta \end{bmatrix} = \int_\alpha^\beta f(t)dt = F(t)\Big|_\alpha^\beta$$

№1590.

$$I = \int_0^5 \frac{dx}{2x + \sqrt{3x+1}} =$$

$$= \begin{bmatrix} t = \sqrt{3x+1} \Rightarrow 3x+1 = t^2 \Rightarrow x = \frac{t^2-1}{3} = \frac{1}{3}t^2 - \frac{1}{3} \\ dx = \frac{2}{3}t\,dt \\ 2x + \sqrt{3x+1} = 2\,\frac{t^2-1}{3} + t^{\llcorner 3} = \frac{2t^2+3t-2}{3} \\ x=0 \Rightarrow t=1\ ;\ x=5 \Rightarrow t=4 \end{bmatrix} =$$

$$\int_1^4 \frac{\frac{2}{3}t\,dt}{\frac{2t^2+3t-2}{3}} = 2\int_1^4 \frac{t\,dt}{2t^2+3t-2} =$$

$$= \begin{bmatrix} (2t^2+3t-2)' = 4t+3 \\ t = \frac{1}{4}\cdot 4t = \frac{1}{4}(4t+3-3) = \frac{1}{4}(4t+3) - \frac{3}{4} \end{bmatrix} =$$

$$= 2\int_1^4 \frac{\frac{1}{4}(4t+3)-\frac{3}{4}}{2t^2+3t-2}\,dt =$$

$$= \frac{1}{2}\underbrace{\int_1^4 \frac{(4t+3)\,dt}{2t^2+3t-2}}_{I_1} - \frac{3}{2}\underbrace{\int_1^4 \frac{dt}{2t^2+3t-2}}_{I_2}$$

$$I_1 = \int_1^4 \frac{d(2t^2+3t-2)}{2t^2+3t-2} = \ln|2t^2+3t-2|\Big|_1^4 =$$

$$= \ln|2\cdot 16+12-2| - \ln|2+3-2| = \ln 42 - \ln 3 =$$

$$= \ln\frac{42}{3} = \ln 14$$

$$I_2 = \begin{bmatrix} 2t^2+3t-2 = 2\left(t^2+\frac{3}{2}t-1\right) = 2\left(t^2+2\cdot\frac{3}{4}t+\frac{9}{16}-\frac{9}{16}-1\right) = \\ = 2\left(\left(t+\frac{3}{4}\right)^2-\frac{25}{16}\right) \end{bmatrix} =$$

$$= \frac{1}{2}\int_1^4 \frac{d\left(t+\frac{3}{4}\right)}{\left(t+\frac{3}{4}\right)^2-\left(\frac{5}{4}\right)^2} = \frac{1}{\cancel{2}}\cdot\frac{1}{\cancel{2}\cdot\frac{5}{\cancel{4}}}\ln\left|\frac{\left(t+\frac{3}{4}\right)-\frac{5}{4}}{\left(t+\frac{3}{4}\right)+\frac{5}{4}}\right|\Bigg|_1^4 =$$

$$= \frac{1}{5}\ln\left|\frac{t-\frac{1}{2}}{t+2}\right|\Bigg|_1^4 = \frac{1}{5}\left(\ln\frac{4-\frac{1}{2}}{4+2} - \ln\frac{1-\frac{1}{2}}{1+2}\right) = \frac{1}{5}\ln\frac{7}{2}$$

$$I=\frac{1}{2}\ln 14-\frac{3}{2\cdot 5}\ln\frac{7}{2}=$$

$$=\frac{1}{2}(\ln 2+\ln 7)-\frac{3}{10}(\ln 7-\ln 2)=$$

$$=\frac{5}{10}\ln 2+\frac{3}{10}\ln 2+\frac{5}{10}\ln 7-\frac{3}{10}\ln 7=$$

$$=\frac{4}{5}\ln 2+\frac{1}{5}\ln 7=\frac{1}{5}(\ln 2^4+\ln 7)=\frac{1}{5}\ln 112.$$

$$\boxed{\text{Д/З V } \quad №1591,\ 1589,\ 1593}$$

№1587

$$\int\limits_{\frac{\sqrt{2}}{2}}^{1}\frac{\sqrt{1-x^2}}{x^2}\,dx=$$

$$=\left[\begin{array}{l} x=\sin t\Rightarrow \dfrac{\sqrt{1-x^2}}{x^2}=\dfrac{\sqrt{1-\sin^2 t}}{\sin^2 t}=\dfrac{\sqrt{\cos^2 t}}{\sin^2 t}= \\ \qquad =\dfrac{|\cos t|}{\sin^2 t}=\dfrac{\cos t}{\sin^2 t}; \\ \qquad \text{т.к. } x\in\left[\frac{\sqrt{2}}{2};1\right]\Rightarrow \cos t>0 \\ \qquad dx=\cos t\,dt \\ x=\dfrac{\sqrt{2}}{2}\Rightarrow t=\dfrac{\pi}{4} \\ x=1\Rightarrow t=\dfrac{\pi}{2} \end{array}\right]=$$

$$=\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos t}{\sin^2 t}\cos t\,dt=-\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos^2 t\,d(\operatorname{ctg} t)=$$

$$=\left[\cos^2 t = \frac{ctg^2 t}{1+ctg^2 t}\right] = -\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{ctg^2 t}{1+ctg^2 t}\, d\,ctg\,t =$$

$$=\left[\begin{matrix} ctg\,t = a \\ t=\frac{\pi}{2} \Rightarrow a=0\ ;\ t=\frac{\pi}{4} \Rightarrow a=1 \end{matrix}\right] =$$

$$= -\int\limits_1^0 \frac{a^2}{1+a^2}\,da = \int\limits_0^1 \frac{a^2}{a^2+1}\,da = \int\limits_0^1 \frac{a^2+1-1}{a^2+1}\,da =$$

$$= \int\limits_0^1 \left(1 - \frac{1}{a^2+1}\right) da = a\Big|_0^1 - arctg\,a\Big|_0^1 =$$

$$= (1-0) - \left(\frac{\pi}{4} - 0\right) = 1 - \frac{\pi}{4}.$$

$\boxed{\text{Д13 } \underline{\text{VI}}. \quad N1592 \quad \underline{\text{Указ}}.\ x = tg\,t.}$

Вычисление определённого интеграла по частям.

$$\int_a^b u(x)\,dv(x) = u(x)v(x)\Big|_a^b - \int_a^b v(x)\,du(x)$$

№1599.

$$\int_0^{\frac{\pi}{2}} \underbrace{x}_{u}\, \underbrace{\cos x\,dx}_{dv} = \int_0^{\frac{\pi}{2}} x\,d\sin x =$$

$$= x\sin x\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\,dx =$$

$$= \left(\frac{\pi}{2}\sin\frac{\pi}{2} - 0\sin 0\right) - (-\cos x)\Big|_0^{\frac{\pi}{2}} =$$

$$= \frac{\pi}{2} + \left(\cos\frac{\pi}{2} - \cos 0\right) = \frac{\pi}{2} + (0-1) = \frac{\pi}{2} - 1$$

№1600

$$\int_1^e \underbrace{\ln x}_{u}\,\underbrace{dx}_{dv} = \ln x\cdot x\Big|_1^e - \int_1^e x\,d(\ln x) =$$

$$= (\ln e\cdot e - \ln 1\cdot 1) - \int_1^e x\cdot\frac{1}{x}\,dx = 1\cdot e - x\Big|_1^e = e-(e-1) = 1$$

$\boxed{\text{Д/З VII №1601, №1602}}$