BAUMAN MOSCOW STATE TECHNICAL UNIVERSITY

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INTRODUCTION INTO STRENGTH OF MATERIALS IN ENGLISH

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PREFACE

These lectures are some kind of compilation mainly based on the original textbook by James M. Gere and Stephen P. Timoshenko.

The lectures cover all standard topics of mechanics of materials. These topics include the analysis and design of structural members subjected to axial loads, torsion and bending, as well as such fundamental concepts as stress, strain, elastic and inelastic behavior, strain energy, and mechanical properties of materials. Specialized topics such as stress concentrations, fatigue and dynamic loading, thermal and prestrain effects, behavior of columns, and pressure vessels are considered, too.

The lectures provide an opportunity for students and engineers to master their technical English and develop their abilities to analyze and discuss different problems related to strength of materials and engineering design.

I am pleased to express my gratitude to professor B. G. Popov who has read all the lectures thoroughly and suggested some valuable ideas.

I appreciate very much a computer version and design of the lectures done by A. E. Andreev-Andrievsky.

S. Andrievskaya

CONTENTS

Page

Lecture 1 5
INTRODUCTION • PROBLEMS OF STRENGTH OF MATERIALS • GENERAL ASSUMPTIONS • BASIC TYPES OF LOADING
Lecture 2 10
CLASSIFICATION OF EXTERNAL FORCES • METHOD OF SECTIONS • STRESS
Lecture 3 15
TENSION, COMPRESSION • NORMAL STRESS • NORMAL STRAIN
Lecture 4 20
HOOKE'S LAW
Lecture 5 24
POISSON'S RATIO • VOLUME CHANGE
Lecture 6 28
EXPERIMENTAL STUDY OF MATERIALS IN TENSION AND COMPRESSION • TENSION TEST • TENSION TEST DIAGRAM • STRUCTURAL STEEL DIAGRAM
Lecture 7 35
DUCTILITY OF MATERIAL • OFFSET YIELD STRESS • BRITTLE MATERIALS
Lecture 8 40
COMPRESSION TEST • COMPRESSION TEST DIAGRAM • CREEP
Lecture 9 45
ELASTICITY AND PLASTICITY • UNLOADING AND RELOADING • STRAIN HARDENING
Lecture 10 49
ALLOWABLE STRESSES AND ALLOWABLE LOADS • EXAMPLE
Lecture 11 54
STRESSES ON INCLINED SECTIONS
Lecture 12 58
STRAIN ENERGY
Lecture 13 63
STRESS CONCENTRATIONS • SAINT-VENANT'S PRINCIPLE
Lecture 14 68
FATIGUE
Lecture 15 72
DYNAMIC LOADING
Lecture 16 77
SUDDENLY APPLIED LOAD • HORIZONTAL IMPACT • EQUIVALENT STIFFNESS • EXAMPLE
Lecture 17 81
STATICALLY INDETERMINATE STRUCTURES
Lecture 18 85
TEMPERATURE EFFECT • EXAMPLE • APPENDIX

Lecture 19 90
PRESTRAINS
Lecture 20 94
NONLINEAR BEHAVIOR • PLASTIC ANALYSIS OF STATICALLY INDETERMINATE SYSTEMS
Lecture 21 101
TORSION • INTRODUCTION • TORSION OF CIRCULAR BARS
Lecture 22 105
ANGLE OF TWIST • SHEAR STRESSES • HOLLOW CIRCULAR BARS
Lecture 23 109
PURE SHEAR
Lecture 24 113
STRAINS IN PURE SHEAR • RELATIONSHIP BETWEEN MODULI OF ELASTICITY *E* AND *G*
Lecture 25 117
STRAIN ENERGY IN PURE SHEAR AND TORSION • NONUNIFORM TORSION
Lecture 26 123
THIN-WALLED TUBES
Lecture 27 127
BENDING • TYPES OF BEAMS • LOADS • SHEAR FORCE AND BENDING MOMENT • RELATIONSHIPS BETWEEN LOADS, SHEAR FORCE AND BENDING MOMENT • SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS
Lecture 28 134
STRESSES IN BEAMS • INTRODUCTION • CURVATURE • NORMAL STRAINS
Lecture 29 139
NORMAL STRESSES IN BEAMS
Lecture 30 143
COMPOSITE BEAMS • SANDWICH BEAMS
Lecture 31 147
DESIGN OF BEAMS • EXAMPLE
Lecture 32 151
BUCKLING AND STABILITY • COLUMNS WITH PINNED ENDS
Lecture 33 158
COLUMNS WITH OTHER SUPPORT CONDITIONS • EFFECTIVE LENGTH OF COLUMNS • COLUMN WITH FIXED ENDS • COLUMN FIXED AT THE BASE AND PINNED AT THE TOP
Lecture 34 165
SPHERICAL AND CYLINDRICAL PRESSURE VESSELS (BIAXIAL STRESS) • SPHERICAL PRESSURE VESSELS • CYLINDRICAL PRESSURE VESSELS • EXAMPLE

VOCABULARY 171

Lecture 1
INTRODUCTION

Mechanics of materials is a branch of applied mechanics that deals with the behavior of solid bodies subjected to various types of loading. This field of study is known by several names, including "strength of materials" and "mechanics of deformable bodies".

Strength of materials is based upon the laws and theorems of theoretical mechanics. However, in theoretical mechanics solids are conventionally considered as absolutely rigid bodies that is as bodies undergoing no change in shape under the action of loads applied to them. Experimental observations show, however, that all solids deform when subjected to forces. External forces (loads) acting on a solid produce internal forces, which resist the external ones. Thus, for instance, if external forces stretch a solid, the internal forces will resist the stretching, there will act forces of mutual attraction between individual particles of the solid. As external forces increase so do internal forces. However, the internal forces in each material can increase only to a certain limit characteristic of this material. The external forces may be so large that the internal forces in a body of given dimensions will not be able to balance them and the body will fracture. In order for structural members and machine parts to sustain the loads acting on them without fracture and appreciable deformations they must be made of a proper material and have the necessary dimensions. These dimensions of structural members are determined by calculations with making use of equations of strength of materials.

PROBLEMS OF STRENGTH OF MATERIALS

For various types of loading strength of materials establishes mathematical relations between external forces, geometric proportions of structural members, the resulting elastic (i.e. internal) forces and deformations.

These relations and the strength characteristics of materials are used to determine the required dimensions of structural members.

In establishing these relations certain assumptions and limitations are made. These assumptions and limitations are necessary because it is impossible to cover all the features of the phenomenon under study as a whole.

GENERAL ASSUMPTIONS

First of all, the material of which the structures are made is considered to be continuous, homogeneous at all points of the body and having the same properties in all directions. The latter property of the materials is called isotropy. Indeed, some structural materials such as cast metals possess high homogeneity (cast iron is an exception in this case). The more homogeneous the material and the more alike its properties in different directions, the closer is the agreement between theoretical and experimental results. In strength of materials experiment and theory are closely interrelated; all theoretical assumptions and conclusions are verified in practice and after their validity is confirmed they are accepted for use.

Strength of materials, as a rule, deals with only those problems of the behavior of bodies under the action of external forces in which the deformations are small compared to the dimensions of the body. This makes it possible to neglect the changes (produced by deformations) in the position of the forces acting on the body.

Strength of materials deals with only simple-shaped bodies. These are bars, plates, and thin-walled shells.

A bar is a body whose length is considerably greater than the transverse dimensions which are of the same order of magnitude. The axis of the bar may be curved or straight line. A bar with a straight axis may be called rod, shaft, beam, and column depending on its purpose.

A plate and a thin-walled shell are bodies whose thickness is considerably smaller than the other two dimensions. Strength of materials deals mainly with bars.

BASIC TYPES OF LOADING

Loading of structural members may be very complex. However, this complex loading can be always represented as consisting of a small number of basic types of loading. Basic types of loading studied in strength of materials are: tension, compression, shear, torsion, bending.

WORDS AND WORD COMBINATIONS

to apply .... применять, прилагать, прикладывать
appreciable .... заметный, ощутимый; поддающийся определению, оценке
to assume .... допускать, предполагать
assumption .... предположение, допущение
axis (pl. axes) .... ось
bar .... брус
basic .... основной
beam .... балка
behavior .... поведение; тех. режим работы
bending .... изгиб
calculation .... расчет, вычисление
to cast .... тех. отливать, лить (металлы)
column .... колонна , столбик, стойка
compared to .... по сравнению с
in comparison with .... по сравнению с
compression .... сжатие
to confirm .... подтверждать
to consider .... рассматривать, обсуждать; учитывать, принять во внимание; полагать, считать
considerable .... значительный
to consist .... состоять (из чего-либо of)
continuity .... непрерывность
continuous .... непрерывный, сплошной
conventional .... условный; общепринятый
to cover .... здесь охватывать
curve .... кривая линия
curved .... искривленный, кривой
to deal (dealt, dealt) .... иметь дело (с кем-л. - with); решать (вопрос, проблему; with)
to decrease .... уменьшать(ся), убывать

to depend .... зависеть (от on, upon)
to determine .... определять, устанавливать
dimension .... размер
equation .... мат. уравнение
to establish .... устанавливать, основывать, доказывать
external .... внешний, наружный
feature .... особенность, характерная черта
to fracture .... ломать(ся)
homogeneity .... однородность; гомогенность
homogeneous .... однородный
however .... однако, тем не менее, не смотря на; как бы ни
to increase .... расти, возрастать; увеличивать(ся), повышать(ся)
internal .... внутренний
isotropy .... изотропия
length .... длина
load .... нагрузка
magnitude .... величина
to make use .... использовать, воспользоваться
mutual .... взаимный
to neglect .... пренебрегать, не обращать внимания
to observe .... наблюдать, замечать; изучать
in order that .... с тем, чтобы
in order to .... для того, чтобы
phenomenon (pl. phenomena) .... явление
plate .... пластина
to possess .... владеть, обладать
problem .... мат. задача; проблема
property .... свойство, качество
purpose .... цель, назначение
relation .... (обыкнов. pl.) отношение, соотношение, связь
to represent .... представлять, изображать

to require .... требовать, нуждаться
required .... требуемый
rigid .... жесткий, негнущийся, твердый
rod .... стержень
shaft .... тех. вал, ось
shape .... форма; очертание, вид
shear .... сдвиг
solid .... твердый; крепкий, прочный; сплошной; целый
solid body .... твердое тело
to solve a problem .... решать задачу
straight .... прямой
strength of materials .... сопротивление материалов
to stretch .... растягивать(ся), вытягивать(ся), натягивать(ся)
structural member .... конструктивный элемент
to subject .... подвергать (воздействию, влиянию)
to sustain .... поддерживать; выдерживать; подвергаться, переносить, испытывать
tension .... растяжение
thickness .... толщина
thin-walled shell .... тонкостенная оболочка
torsion .... кручение
transverse .... поперечный
to undergo (underwent, undergone) .... подвергаться, испытывать, переносить
to use .... употреблять, применять, пользоваться
validity .... действительность, законность
various .... различный, разный; разнообразные; (перед pl.) многие
to verify .... проверять; подтверждать; удостоверять

Lecture 2

CLASSIFICATION OF EXTERNAL FORCES

External forces (loads) may act on machine and structural parts in different ways. According to the manner in which they are applied forces may be divided into body and surface forces. Among body forces is, for example, the gravity force (weight). Surface forces are divided into distributed and concentrated ones. Distributed forces are applied over an area or along a length.

The force distributed over an area is expressed in units of force per unit area (N/m^2 = Pa, MPa). A load distributed along a length is expressed in units of force per unit length (N/m, kN/m).

Concentrated forces act over a very small area. A concentrated force is considered to be applied at a point for the sake of simplicity; this simplification introduces no serious error in calculations, as a rule. Concentrated forces are measured in units of force (N, kN).

According to their nature of action loads are divided into static and dynamic ones. A static load is defined as a load, which increases slowly from zero to a certain maximum value and then remains constant or varies only slightly.

An example of dynamic loads is an impact load when the time duration of the load is a small fraction of a second. Dynamic loads also include periodic loads varying in time and inertia forces developed during vibration.

Materials resist these types of forces (static and dynamic) in different ways.

METHOD OF SECTIONS

External forces acting on a body deform it and give rise to internal resisting forces. The internal forces are determined by the method of sections. The idea of this method is as follows.

Consider a body, which is in a state of equilibrium under the action of forces P_1, P_2, P_3, P_4 (Fig.1).

Imagine the body cut through the section *a-a* and one of the two parts removed, say, the right-hand part. The remaining left-hand part will then be acted on by the external forces P_1 and P_2. In order for this part of the body to remain in equilibrium, it is necessary to apply internal forces over the entire section.

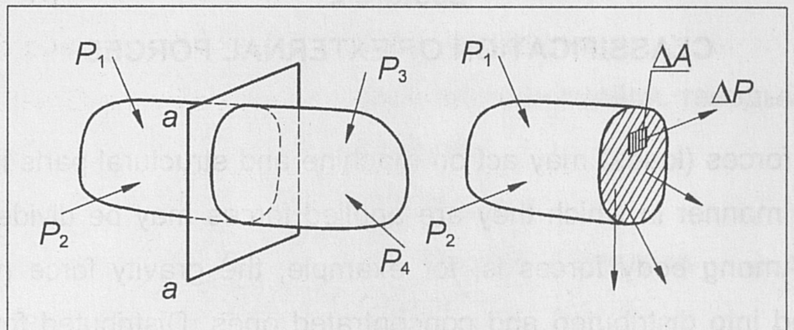

Fig.1

These forces represent the action of the removed right-hand part of the body on the remaining left-hand part. Being internal forces for the entire body, they play the role of external forces for the isolated part. The magnitude of the resultant of the internal forces can be determined from the condition of equilibrium of the isolated part or free body. The law of distribution of internal forces over the sections is not in general known. Thus, the method of sections only allows us to determine the sum of the internal forces acting at the section in question. The sum of these forces may be reduced, in the general case, to a force and a couple which are called stress resultants.

STRESS

If an infinitesimal area ΔA is acted on by an infinitesimal force ΔP (Fig.1) the ratio of the force ΔP to the area ΔA gives the average stress on this area

$$p_{AV} = \frac{\Delta P}{\Delta A}$$

p_{AV} is equal to the ratio of ΔP to ΔA or is equal to ΔP divided by ΔA

The stress is expressed in newtons per square meter (N/m^2 = Pa) or MPa, where 1 MPa = 10^6 N/m^2 (ten to the sixth power). Reducing the area to zero that is passing to the limit we obtain the true stress at a given point:

$$p_{TRUE} = \lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A} = \frac{dP}{dA}$$

$\Delta A \to 0$: ΔA tends to zero

Since the force has a direction, the stress will also have a direction.

Resolve this stress into two components (Fig.2), one being perpendicular to the area, called the normal stress and denoted by the Greek letter σ, and the other lying in the plane of the section, called the shear stress and denoted by the Greek letter τ.

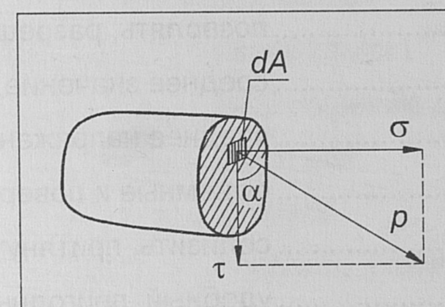

Fig.2

The total stress is expressed in terms of the normal and shear stresses by the following equation

$$p = \sqrt{\sigma^2 + \tau^2}$$

p is equal to the square root (out) of σ square plus τ square

The total stress is not considered to be a convenient measure of internal forces in a body as materials resist normal and shear stresses in different ways.

Normal stresses tend to bring closer together or separate individual particles of a body in the direction of the normal to the plane of the section.

Shear stresses tend to move particles of a body with respect to each other on the plane of the section.

WORDS AND WORD COMBINATIONS

according to .... согласно
in accordance with .... согласно, в соответствии с
to allow .... позволять, разрешать; допускать
average... среднее значение, средняя величина
average stress .... среднее напряжение
body and surface forces .... объемные и поверхностные силы
to bring closer together .... сблизить, притянуть друг к другу
convenient .... удобный, пригодный, подходящий
couple .... пара; тех. пара сил
to denote... обозначать
direction .... направление
to distribute .... распределять
to divide .... делить(ся), разделять(ся)
entire .... цельный; сплошной; полный
equal... равен
to equal... равняться
expressed in terms of .... выраженное через что-либо, в чем-либо
is as follows .... состоит в следующем
free body... отрезанная часть тела с действующими на нее внешними и внутренними силами, приложенными к сечению
to give rise to .... вызывать что-либо; порождать
to imagine .... представлять, воображать, полагать
impact .... удар, столкновение
to include .... заключать; включать; содержать (в себе)
infinitesimal... бесконечно малый
manner .... способ; образ действий; манера
to measure... мерить, измерять
to obtain .... получать, добиваться, достигать
to occur... иметь место, случаться
per .... на
plane... плоскость

ratio .... отношение, пропорция, соотношение
to reduce... приводить в определенное состояние; уменьшать; сокращать
to remain... оставаться
to resolve .... разлагать(ся)
with respect to each other .... по отношению друг к другу
for the sake of.. ради, для
section .... сечение
to separate... отделить, разъединить
shear stress .... касательное напряжение
slightly.. слегка
state of equilibrium .... состояние равновесия
stress resultants .... равнодействующие напряжений (внутренние суммарные силовые факторы)
to tend .... стремиться
total.. полный, весь, целый, общий; суммарный, совокупный
unit.. единица; единица измерения
unity .... мат. единица
to vary... менять(ся), изменять(ся), отличаться

Lecture 3

TENSION, COMPRESSION. NORMAL STRESS

Consider a prismatic bar of a constant cross-sectional area that is loaded by axial forces P at the ends (Fig.1). These equal and opposite forces are applied in such a way that they will act precisely along the axis of the bar. The bar being in equilibrium under the action of the tensile forces will elongate in the longitudinal direction and its transverse dimensions will contract.

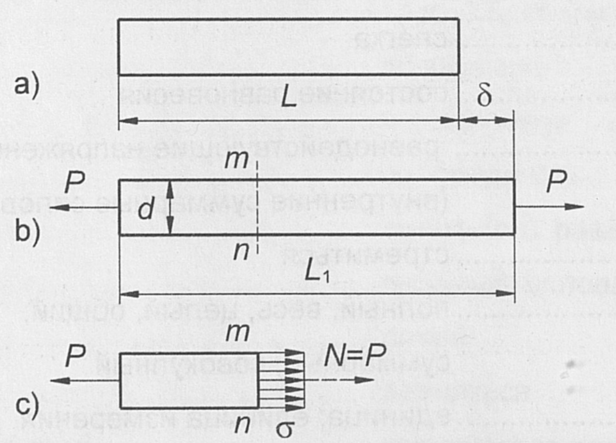

Fig.1

We shall assume that all plane sections normal to the axis of the bar remain plane and normal to its axis after deformation. This hypothesis is known as the hypothesis of plane sections. It is supported by experimental evidence for sections sufficiently far removed from the point of application of the force P. By accepting this hypothesis it is assumed that all longitudinal elements of the bar are stretched in the same manner. The axial forces produce a uniform stretching of the bar, and the bar is said to be in tension. To determine internal stresses produced in the bar by the axial forces we apply the method of sections. Imagine the bar cut into two parts at section m-n (Fig.1b). Because this section is taken perpendicular to the longitudinal axis of the bar it is called a cross section. We now isolate the part of the bar to the left of the cut as a free body (Fig.1c). The tensile load P acts at the left-hand end of this free body; at the other end are forces representing the action of the removed part of the bar upon the part that remains. These forces are continuously distributed over the cross section.

Assuming that the stress has a uniform distribution over the cross section, we see that its resultant N is equal to the intensity σ times the cross-sectional area A of the bar $N = \sigma A$.

- the resultant N is equal to the product of the stress σ multiplied by the cross-sectional area A
- N is equal to σ multiplied by A
- N is equal to σ by A
- N equals σ times A

Furthermore, from the equilibrium of the body it is evident that the resultant N must be equal in magnitude and opposite in direction to the applied load P. From these observations we obtain

$$\sigma = \frac{N}{A} = \frac{P}{A}$$

When the bar is stretched by the forces P, as shown in the figure, the resulting stresses are tensile stresses; if the forces are reversed in direction, we obtain compressive stresses. As the stress σ acts in a direction perpendicular to the cut surface, it is referred to as a normal stress.

When sign convention for normal stresses is required, it is customary to define tensile stress as positive and compressive stress as negative.

Stress has units of newtons per square meter [N/m^2] or pascals [Pa] and [MPa].

If the equation $\sigma = N/A$ is to be valid, the stress σ must be uniformly distributed over the cross section of the bar. This condition is realized if the axial force P acts through the centroid of the cross-sectional area. When the load P does not act at the centroid, bending of the bar will result.

The uniform stress condition exists throughout the length of the bar except near the ends where loads are applied. The load usually is concentrated over a small area, resulting in high localized stresses (called stress concentrations) and nonuniform stress distributions over a cross section in the vicinity of the load. As we move away from the ends, the stress distribution gradually approaches the uniform distribution. It is usually safe to assume that the formula $\sigma = N/A$ may be used with good accuracy at any point within the bar that is at least a distance d away from the ends, where d is the largest transverse dimension of the bar.

NORMAL STRAIN

An axially loaded bar undergoes a change in length, becoming longer when in tension and shorter when in compression. The change in length is denoted by the Greek letter δ (Fig.1b).

This increment in the length of the bar is called the total or absolute elongation in tension; in the case of compression it is called the total or absolute contraction

$$\delta = L_1 - L$$

the total elongation δ is equal to the difference between the final length L_1 of the bar and its original length L

The absolute elongation depends obviously on the original length of the bar. Therefore, a more convenient measure of deformation is elongation per unit length, or strain, denoted by the Greek letter ε and given by the equation

$$\varepsilon = \frac{\delta}{L}$$

ε is equal to the ratio of δ to L

If the bar is in tension, the strain is called a tensile strain, representing elongation or stretching of the material. If the bar is in the compression the strain is a compressive strain and the bar shortens. Tensile strain is usually taken as positive and compressive strain as negative. The strain ε is called a normal strain because it is associated with normal stresses.

Normal strain ε is a dimensionless quantity, it has no units, it is a pure number.

Numerical values of strain are usually very small, especially for structural materials, which ordinarily undergo only small changes in dimensions. As an example, consider a steel bar having length L equal to 2.0 m. When loaded in tension, the bar might elongate by an amount δ equal to 1.4 mm. The corresponding strain is

$$\varepsilon = \frac{\delta}{L} = \frac{1.4 \times 10^{-3}\ \text{m}}{2.0\ \text{m}} = 0.0007 = 7.0 \cdot 10^{-4}.$$

- o point o o o seven
- zero point zero zero zero seven
- point three oes seven
- point three nougths seven
- seven times ten to the minus fourth power

WORDS AND WORD COMBINATIONS

accuracy .... точность, тщательность
amount .... количество
to approach .... приближаться
axial force .... осевая сила
the bar is in tension .... брус работает на растяжение
centroid .... центр тяжести сечения
to contract .... сокращать(ся), сжимать(ся), суживать(ся)
contraction .... укорочение
corresponding .... соответствующий, соответственный
cross-section .... поперечное сечение
customary .... обычный, привычный
to define .... определять, давать определение
to elongate .... растягивать, удлинять
elongation .... удлинение
evident .... явный, очевидный, ясный
furthermore .... кроме того, более того, к тому же
hypothesis (pl. hypotheses) .... гипотеза
increment .... увеличение, прирост
at least a distance *d* away from .... не менее, чем на расстоянии *d* от
longitudinal .... продольный
nonuniform distribution .... неравномерное распределение
ordinarily .... обычно
original .... первоначальный

perpendicular .... перпендикулярный
the point of application .... точка приложения
precisely .... точно, как раз
to produce .... производить
quantity .... количество; мат. величина
to refer .... отсылать (к - to);
относиться, иметь отношение
to remove .... удалять, перемещать
to shorten .... укорачивать(ся), сокращать(ся)
sign convention .... правило знаков
strain .... деформация
sufficiently .... достаточно
tensile .... растягивающий
σ times the area A .... σ, умноженная на площадь A
uniform .... равномерный
valid .... обоснованный, имеющий силу
value .... величина, значение
in the vicinity .... поблизости (от - of), в окрестности, вблизи

Lecture 4
HOOKE'S LAW

The famous English scientist Robert Hooke (1635 - 1703) established the linear relationship between the applied load and the resulting elongation (1678). For a bar in tension or compression, Hooke's law expresses direct proportionality between stress and strain

$$\sigma = E\varepsilon \qquad (1)$$

σ is equal to the product of *E* times ε or σ is equal to *E* by ε

This proportionality (or linear relationship) is violated when the stress exceeds a certain limit called the proportional limit. The proportional limit for material is established by experiment.

In Eq.(1) *E* is a constant of proportionality known as the modulus of elasticity for the material. The units of *E* are the same as the units of stress, inasmuch as strain is dimensionless, typical units of *E* are MPa in SI units, or GPa. The modulus of elasticity is often called Young's modulus, after another English scientist, Thomas Young (1773-1829) who introduced it into the science.

The magnitude of the modulus of elasticity is established experimentally.

Moduli of elasticity:

steel	$E \cong 200$ GPa,	cast iron	$E \cong 100$ GPa,
copper	$E \cong 100$ GPa,	aluminum	$E \cong 70$ Gpa,

symbol G means multiplication factor 10^9.

Modulus of elasticity characterizes the stiffness of the material, that is its ability to resist deformation, that follows from the equation

$$\varepsilon = \frac{\sigma}{E} \qquad (2)$$

For one and the same stress, the strain will be smaller for the material for which *E* is larger. Formula that expresses Hooke's law may be written in an alternate form. Substituting the appropriate expressions for $\sigma = N/A$ and $\varepsilon = \delta/L$ we obtain the equation for the elongation of the bar

$$\delta = \frac{NL}{EA} \qquad (3)$$

From the formula, it follows that the elongation (contraction) of the bar is directly proportional to the length of the bar, and inversely proportional to the cross-sectional area and the modulus of elasticity. The product EA in the denominator of Eq.(3) is known as the axial rigidity of the bar.

The stiffness of the bar K is defined as the force required to produce a unit elongation

$$K = \frac{N}{\delta} \quad \text{or} \quad K = \frac{EA}{L}$$

The flexibility f is defined as the elongation due to a unit load

$$f = \frac{\delta}{N} \quad \text{or} \quad f = \frac{L}{EA}$$

The flexibility is the reciprocal of the stiffness

$$f = \frac{1}{K}$$

Equation (3) can be adapted to handle more general situation.

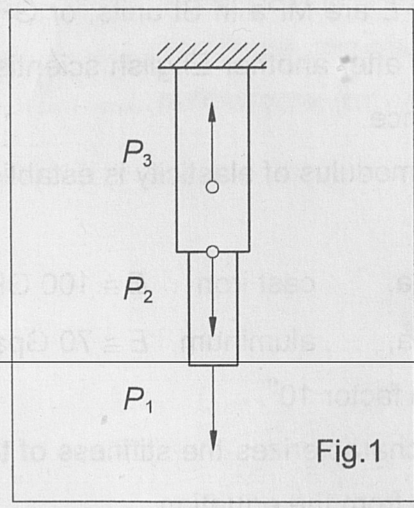

Fig.1

Suppose, for instance, that a prismatic stepped bar (Fig.1) is loaded by some number of intermediate axial loads. We can determine the axial force in each part of the bar by statics, then calculate the elongation or shortening of each part separately. Finally, these changes in length can be added algebraically to obtain the change in the length of the entire bar.

In general, the total elongation δ of a bar consisting of several prismatic parts having different axial forces, dimensions, or materials may be obtained from equation

$$\delta = \sum_{i=1}^{n} \frac{N_i L_i}{E_i A_i}$$

δ is equal to the sum from i equals one to i equals n of capital N sub i times capital L sub i all over capital E sub i by capital A sub i

in which the subscript i is a numbering index for the various parts of the bar and n is the total number of parts. N_i is an axial force in part i (internal force).

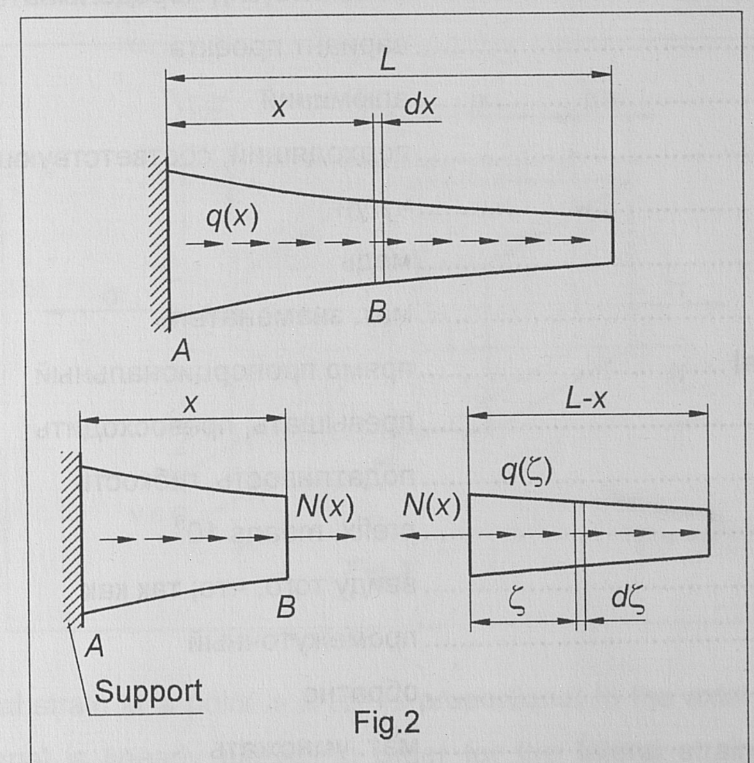

Fig.2

If either the axial force or the cross-sectional area varies continuously along the axis of the bar (Fig.2) the elongation of the entire bar is obtained by integrating over the length the expression for the elongation of a differential element of a bar

$$\delta = \int_0^L d\delta = \int_0^L \frac{N(x)}{E\,A(x)}\,dx$$

δ is equal to the integral of N of x multiplied by dx divided by E multiplied by A of x, from 0 to L

The force $N(x)$ at the cross section at distance x from the support is known in terms of x (Fig.2)

$$N(x)=\int_{x}^{L-x} q(\zeta)\,d\zeta$$

Also, the cross-sectional area $A(x)$ at that section may be expressed as a function of x.

WORDS AND WORD COMBINATIONS

to adapt	приспособить
to add	прибавлять; присоединять, дополнять
to alter	изменять(ся); переделывать
alternate design	вариант проекта
aluminum	алюминий
appropriate	подходящий, соответствующий
cast iron	чугун
copper	медь
denominator	мат. знаменатель
directly proportional	прямо пропорциональный
to exceed	превышать, превосходить
flexibility	податливость, гибкость
giga	prefix, means 10^9
inasmuch as	ввиду того, что; так как
intermediate	промежуточный
inversely	обратно
to multiply	мат. умножать
product	мат. произведение
proportional limit	предел пропорциональности
reciprocal	обратная величина
relationship	связь, соотношение
steel	сталь
stepped bar	ступенчатый брус
stiffness	жесткость
to substitute	заменять, замещать
variable	мат. переменная величина; изменчивый; переменный; непостоянный
to violate	нарушать (закон и т.д.)

Lecture 5

POISSON'S RATIO

When a prismatic bar is loaded in tension, the axial elongation is accompanied by lateral contraction (normal to the direction of the applied load). This change in shape is pictured in Fig.1, in which the dashed lines represent the shape before loading and the solid lines give the shape after loading. In metals the changes in lateral dimensions are usually too small to be visible. However, they can easily be detected with measuring devices.

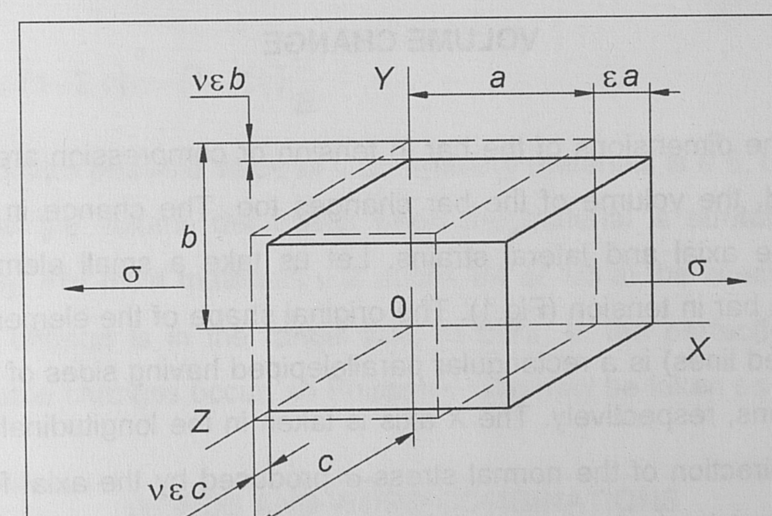

Fig.1

The lateral strain at a point in a bar is proportional to the axial strain at the same point if the material is linearly elastic. In order for the lateral strains to be the same throughout the bar, the material must be homogeneous and isotropic.

The absolute value of the ratio of the strain in the lateral direction to the strain in the axial direction is known as *Poisson's ratio* and denoted by the Greek letter ν, thus

$$\nu=\left|\frac{\text{lateral strain}}{\text{axial strain}}\right|$$

Poisson's ratio is named for the famous French mathematician Simeon Denis Poisson (1781 - 1840), who attempted to calculate this ratio by a molecular theory of materials. For many metals the value of Poisson's ratio is in the range 0.25 to 0.35. For all materials ν is in the range 0.0 to 0.5. Poisson's ratio normally is established experimentally. Materials with extremely low value of Poisson's ratio include cork, for which ν is practically zero, and concrete for which ν is about 0.1 or 0.2. Rubber comes close to the upper limit for Poisson's ratio that equals 0.5.

Poisson's ratio for some materials:

steel ν = 0.27-0.30, cast iron ν = 0.20-0.30,

aluminum ν = 0.33, pure copper ν = 0.33-0.36

The lateral contraction of a bar in tension, or the lateral expansion of a bar in compression is an illustration of how a normal strain can exist in a certain direction without a normal stress acting in the same direction.

VOLUME CHANGE

Because the dimensions of the bar in tension or compression are changed when the load is applied, the volume of the bar changes too. The change in volume can be calculated from the axial and lateral strains. Let us take a small element of isotropic material cut from a bar in tension (Fig.1). The original shape of the element (shown in the figure by the dashed lines) is a rectangular parallelepiped having sides of length *a*, *b*, *c* in the *X, Y, Z* directions, respectively. The *X* axis is taken in the longitudinal direction of the bar, which is the direction of the normal stress σ produced by the axial forces. The final shape of the element is shown by the solid lines. The elongation of the element in the direction of loading is εa, where ε is the axial strain. Because the lateral strains are $-\varepsilon\nu$, the lateral dimensions decrease by $\nu\varepsilon b$ and $\nu\varepsilon c$ in the *Y* and *Z* directions, respectively. Thus, the final dimensions of the element are $(1+\varepsilon)\,a$, $(1-\nu\varepsilon)\,b$ and $(1-\nu\varepsilon)\,c$, and the final volume is

$$V_F = (1+\varepsilon)(1-\nu\varepsilon)(1-\nu\varepsilon)\,a\,b\,c$$

> Round brackets opened one plus ε round brackets closed,
> open round brackets one minus ν times ε, close the round brackets,
> parentheses one minus $\nu\varepsilon$, close parentheses, *abc*.

When the expression is expanded, we obtain terms involving the square and cube of ε. Because ε is very small compared to unity, its square and cube are negligible in comparison with ε itself and may be dropped from the expression. Therefore, the final volume of the element is

$$V_F = (1+\varepsilon-2\,\nu\,\varepsilon)\,a\,b\,c$$

and the change in volume is

$$\Delta V = V_F - V_0 = (1-2\,\nu)\,\varepsilon\,V_0 = (1-2\,\nu)\,\frac{\sigma}{E}\,V_0\,,$$

where $V_0 = a\,b\,c$ is the original volume.

The unit volume change Θ is defined as the change in volume divided by the original volume

$$\Theta = \frac{\Delta V}{V_0} = (1-2\,\nu)\,\varepsilon = (1-2\,\nu)\,\frac{\sigma}{E}$$

The maximum possible value of ν for ordinary materials is 0.5, because any larger value means that the volume decreases when the material is stretched, which seems physically unlikely. For most materials ν is about 1/4 or 1/3 in the linear elastic region, so the unit volume change is in the range 0.3ε to 0.5ε. In the perfectly plastic region of behavior, no volume changes occur, so Poisson's ratio may be taken as 0.5.

WORDS AND WORD COMBINATIONS

concrete .... бетон
cork .... пробка
dashed line .... пунктирная линия
to drop .... (здесь) опустить
to expand .... мат. раскрывать формулу
lateral .... поперечный, боковой
measuring device .... измерительный прибор, приспособление
negligible .... незначительный,
не принимаемый во внимание
original .... первоначальный
parallelepiped .... параллелепипед
range .... зона, область
in the range *a* to *b* .... в диапазоне, в пределах от *a* до *b*
rectangular........ прямоугольный

respectively .... соответственно
rubber .... резина
term .... мат. член
unit volume change .... относительное изменение объема
unlikely .... маловероятно
volume .... объем
( ) round brackets; parentheses .... круглые скобки
[ ] square brackets; brackets .... квадратные скобки
{ } curly brackets; braces .... фигурные скобки

Lecture 6

EXPERIMENTAL STUDY OF MATERIALS IN TENSION AND COMPRESSION

The mechanical properties of materials used in engineering are determined by tests performed on small specimens of the material. The tests are conducted in materials-testing laboratories equipped with testing machines capable of loading the specimens in a variety of ways, including static and dynamic loading in tension and compression.

In order that test results will be comparable, the dimensions of test specimens and the methods of applying loads have been standardized.

TENSION TEST

The ASTM [1] standard tension specimen is a cylindrical specimen (Fig.1) that has a diameter of 12.8 mm and a gage length of 50.8 mm. Gage length is a distance between the gage marks, which are the points where the extensometer arms are attached to the specimen.

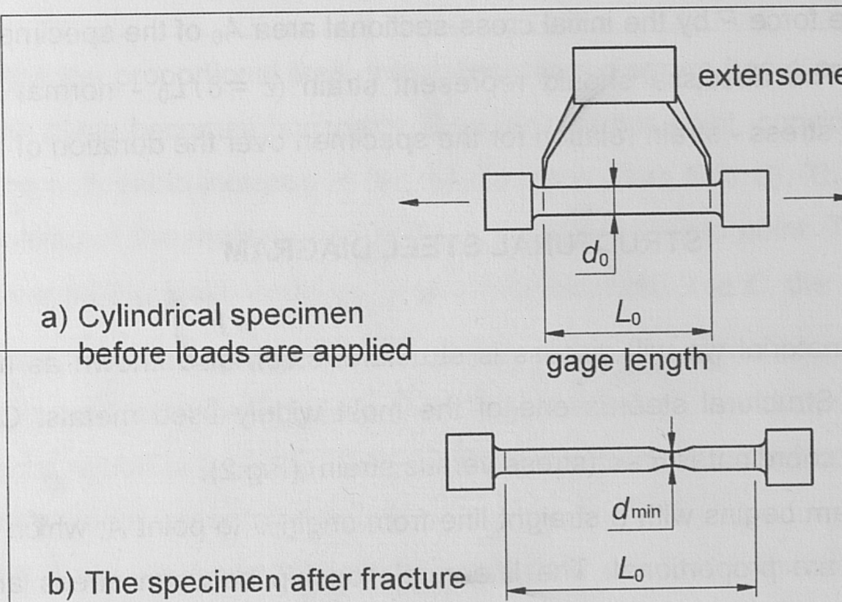

Fig.1

[1] ASTM - the American Society for Testing and Materials, one of the major standards organizations.

As the specimen is pulled, the load *P* is measured and recorded, either automatically or by reading from a dial. The elongation over the gage length is measured simultaneously, either by mechanical gages of the kind shown in Fig.1a or by electrical-resistance strain gages. In a static test, the load is applied very slowly. In a dynamic test the rate of loading may be very high and must be measured because it affects the properties of the materials.

TENSION TEST DIAGRAM

The behavior of materials in tension is best understood from a consideration of a curve called a tension test diagram. It is usually obtained from a diagram in the coordinates: tensile force *P* and absolute elongation of a specimen δ. A P-δ diagram may be traced by a recording instrument.

A diagram of these coordinates will, of course, depend on the dimensions of a specimen. In order to make these diagrams independent of the dimensions of test pieces and comparable for different materials, the ordinate should represent stress σ obtained by dividing the tensile force *P* by the initial cross-sectional area A_0 of the specimen ($\sigma = P / A_0$ - normal stress). The abscissa should represent strain ($\varepsilon = \delta / L_0$ - normal strain). This diagram gives the stress - strain relation for the specimen over the duration of the test.

STRUCTURAL STEEL DIAGRAM

The first material we will discuss is structural steel, also known as mild steel or low-carbon steel. Structural steel is one of the most widely used metals. Consider the diagram plotted in coordinates σ - ε (stress versus strain) (Fig.2).

The diagram begins with a straight line from origin 0 to point *A*, which means that stress and strain are proportional. The linear relationship between stress and strain in simple tension and compression is expressed by the equation $\sigma = E\varepsilon$ that is known as Hooke's law. This straight line makes a very small angle with the axis of ordinates, i.e., the elongations of the specimen increase slowly in this portion. Beyond point *A*, the proportionality between stress and strain no longer exists; hence, the stress at *A* is called the proportional limit. For low-carbon steel this limit is in the range 200 to 280 MPa.

However, high-strength steels can have proportional limits of 550 MPa and more. The slope of the straight line from 0 to *A* is the modulus of elasticity.

With the proper use of scales, we can obtain the modulus of elasticity as the ratio $E = \sigma / \varepsilon$.

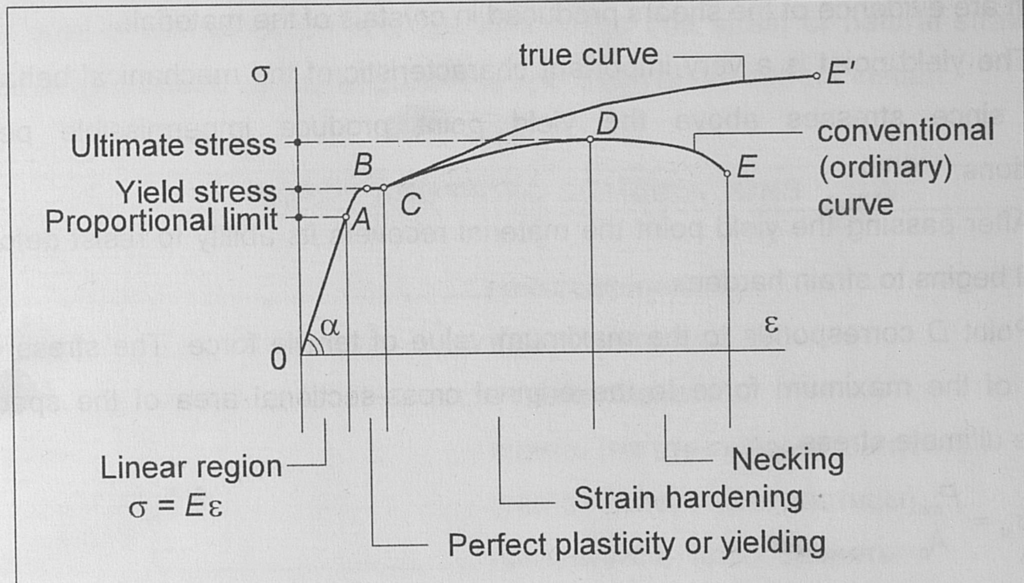

Fig.2

Beyond the proportional limit, the stress-strain diagram has a smaller slope, until, at point *B*, the curve becomes horizontal. Beginning at this point, considerable elongation occurs with no noticeable increase in the tensile force (from *B* to *C*). This phenomenon is known as yielding of the material and point *B* is called the yield point. The corresponding stress is known as the yield stress (σ_y). In the region from *B* to *C*, the material becomes perfectly plastic, which means that it can deform without an increase in the applied load. The material is said to undergo plastic flow. The elongation of mild steel specimen in the perfectly plastic region is typically 10 to 15 times the elongation that occurs between the onset of loading and the proportional limit.

When materials having a pronounced yield point are stretched, it is easy to observe the onset of yielding. If, for example, a tension testing machine is provided with a pointer indicating tensile forces, the pointer stops moving and remains at the same division for some time when the yield point of the material is reached though the deformations of the specimen continue to grow.

The onset of yielding in the material can be noticed also by observing the specimen itself. The polished surface of the specimen dulls and gradually becomes lustreless when the yield point is reached. Under close examination the surface exhibits lines inclined at about 45° to the axis of the specimen. The number of these lines, known as Lüders' lines, increases gradually and eventually the surface of the specimen becomes dull. The occurrence of these lines and their propagation throughout the length of the specimen are evidence of the shears produced in crystals of the material.

The yield point is a very important characteristic of the mechanical behavior of a material since stresses above the yield point produce impermissible permanent deformations.

After passing the yield point the material recovers its ability to resist deformation. The steel begins to strain harden.

Point *D* corresponds to the maximum value of tensile force. The stress equal to the ratio of the maximum force to the original cross-sectional area of the specimen is called the ultimate stress

$$\sigma_u = \frac{P_{max}}{A_0}$$

The ultimate stress is a very important strength characteristic of a material, particularly important for brittle materials, such as cast iron, hardened and cold-drawn steel, which undergo relatively small deformations at fracture.

After the ultimate stress is reached, a local reduction of area of the specimen, called necking, begins to occur gradually. During necking the specimen elongates mainly at the necked-down portion while the remainder of the specimen elongates only slightly. Since during necking the cross section at the neck becomes smaller and smaller, the deformation of the specimen occurs with decreasing load.

At a stress corresponding to point *E* the specimen ruptures. The stress at rupture is below the ultimate stress in the tension test diagram. This is due to the fact that we agreed to calculate the stresses on the basis of the original cross-sectional area of the specimen. Actually, at the time of rupture the material develops the maximum stress since the area of the necked section becomes a minimum at that time (Fig.1b).

This stress (the fracture stress) is called the true ultimate stress

$$\sigma_{TRUE} = \frac{P_E}{A_{min}}, \quad \text{where} \quad A_{min} = \frac{\pi d^2_{min}}{4}$$

The diagram considered above is termed an ordinary or conventional stress-strain diagram since the stresses are related to the original cross-sectional area and the

elongations to the original length. The cross section and length of the specimen vary continuously during the test. However, the ordinary diagram closely coincides with the true one up to the yield point. In the true diagram (curve *ABE'* in Fig.2) the ordinate is the stress obtained by dividing the force by the corresponding value of the minimum cross-sectional area of the specimen and abscissa is the true strain or natural strain, i.e. the change in length divided by the length of the specimen at the current instant.

WORDS AND WORD COMBINATIONS

actual .... действительный
actually .... в действительности
to affect .... воздействовать,
влиять (на что-либо, кого-либо);
(употребляется без частицы)
to attach .... прикреплять, присоединять
beyond .... сверх, выше; за, вне
brittle .... хрупкий
to coincide .... совпадать, соответствовать
to conduct .... проводить
to correspond .... соответствовать (чему-либо with, to)
device .... прибор, приспособление
dial .... циферблат, круглая шкала
division .... деление
to dull .... делать(ся) тусклым
duration .... время действия, продолжительность
electrical-resistance strain gage .... тензодатчик
to equip .... снабжать, обеспечивать
eventually .... в конце концов, в итоге
to exhibit .... проявлять, показывать
extension .... вытягивание
extensometer .... тензометр
failure .... разрушение, авария
fracture .... разрушение; трещина, излом

gage .... мера, размер, шаблон; измерительный прибор
gradually .... постепенно
hence .... отсюда, следовательно
impermissible .... непозволительный
to incline .... наклонять(ся)
initial .... (перво)начальный
low-carbon steel .... малоуглеродистая, мягкая сталь
lustreless .... тусклый, матовый
mild steel .... малоуглеродистая, мягкая сталь
necking .... образование шейки
occurrence .... появление, возникновение
onset .... начало
origin .... начало, источник, происхождение
to perform .... выполнять
permanent .... постоянный, неизменный; остаточный,
plastic flow .... пластическое течение
to plot .... составлять чертеж, план; наносить на чертеж
pointer .... стрелка; указатель
pronounced .... явный, ярко выраженный
propagation .... распространение
to provide .... обеспечивать, снабжать, предусматривать
rapidly .... быстро
rate .... темп, скорость
to recover .... тех. восстанавливать, обретать снова
to reduce .... уменьшать, сокращать
reduction .... уменьшение, сокращение
to relate .... связывать, устанавливать связь; относиться
rupture .... разрыв
to rupture .... разрывать
scale .... масштаб, шкала
simultaneously .... одновременно
slope .... наклон, склон; тангенс угла наклона

specimen .... образец
strain hardening .... упрочнение
stress versus strain .... в координатах σ–ε
structural steel .... малоуглеродистая, мягкая сталь
test .... испытание
a is 10 times *b* .... *a* в 10 раз больше *b*
true .... истинный
true ultimate stress σ_{TRUE} .... напряжение при разрыве $\sigma_{РАЗ}$
ultimate stress σ_u .... (условный) предел прочности, временное сопротивление $\sigma_В$
yield stress σ_y .... предел текучести $\sigma_Т$