

Модуль 1 «Операционное исчисление. Случайные события».
Домашнее задание

Часть 1. Операционное исчисление

Задача 1. ЛИНЕЙНОЕ ДИФФЕРЕНЦИАЛЬНОЕ УРАВНЕНИЕ (2 балла)

Проинтегрировать линейное дифференциальное уравнение при заданных начальных условиях.

1. $x'' + 4x = e^t$; $x(0) = x_0$, $x'(0) = x'_0$.
2. $x'' + 9x = \cos 3t$; $x(0) = x_0$, $x'(0) = x'_0$.
3. $x'' - 4x = t$; $x(0) = x_0$, $x'(0) = x'_0$.
4. $x'' - 9x = \operatorname{sh} 3t$; $x(0) = x_0$, $x'(0) = x'_0$.
5. $x'' - 3x' = t$; $x(0) = x_0$, $x'(0) = x'_0$.
6. $x'' - 4x' + 4x = e^{2t}$; $x(0) = x_0$, $x'(0) = x'_0$.
7. $x'' - 4x' + 5x = e^t$; $x(0) = x_0$, $x'(0) = x'_0$.
8. $x'' + 2x' + 2x = t^2$; $x(0) = x_0$, $x'(0) = x'_0$.
9. $x'' + 2x' + x = e^{-t}$; $x(0) = x_0$, $x'(0) = x'_0$.
10. $x'' + 4x' + 4x = e^{-2t}$; $x(0) = x_0$, $x'(0) = x'_0$.
11. $x'' + x' = e^{-t}$; $x(0) = x_0$, $x'(0) = x'_0$.
12. $x'' + 3x' + 2x = e^t$; $x(0) = x_0$, $x'(0) = x'_0$.
13. $x'' + x' - 2x = e^t$; $x(0) = x_0$, $x'(0) = x'_0$.
14. $x'' - x' - 2x = t$; $x(0) = x_0$, $x'(0) = x'_0$.
15. $x'' - 2x' = e^{2t}$; $x(0) = x_0$, $x'(0) = x'_0$.
16. $x'' + 2x = t$; $x(0) = x_0$, $x'(0) = x'_0$.
17. $x'' + 2x' + x = e^{-t}$; $x(0) = 1$, $x'(0) = 0$.
18. $x'' - 3x' = e^{3t}$; $x(0) = 0$, $x'(0) = -1$.
19. $x'' - 2x' + 2x = \sin t$; $x(0) = 0$, $x'(0) = 1$.
20. $x'' + 4x = \sin 2t$; $x(0) = 1$, $x'(0) = -2$.
21. $x'' - 9x = \operatorname{sh} t$; $x(0) = -1$, $x'(0) = 3$.
22. $x'' + x' = t^2$; $x(0) = 1$, $x'(0) = 0$.
23. $x'' + x' - 2x = e^{-t}$; $x(0) = 1$, $x'(0) = -2$.
24. $x'' - x' - 6x = e^{-t}$; $x(0) = 0$, $x'(0) = -1$.
25. $x''' - x' = t$; $x(0) = 0$, $x'(0) = 1$, $x''(0) = 0$.
26. $x''' - x' = e^t$; $x(0) = 1$, $x'(0) = 0$, $x''(0) = 0$.
27. $x^{\text{IV}} - x = 1$; $x(0) = 1$, $x'(0) = 0$, $x''(0) = 0$, $x'''(0) = 0$.
28. $x^{\text{IV}} - x'' = \operatorname{sh} t$; $x(0) = 0$, $x'(0) = 0$, $x''(0) = 0$, $x'''(0) = 1$.
29. $x^{\text{IV}} - x''' = e^t$; $x(0) = 0$, $x'(0) = 0$, $x''(0) = 0$, $x'''(0) = 1$.
30. $x''' - 2x'' + x' = 1$; $x(0) = 0$, $x'(0) = 0$, $x''(0) = 0$.

Задача 2. СИСТЕМА ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ (2 балла)

Проинтегрировать систему линейных дифференциальных уравнений при заданных начальных условиях.

$$1. \begin{cases} x'' - y' = t, \\ y'' - x' = 0; \end{cases} \quad x(0) = -1, \quad x'(0) = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

$$2. \begin{cases} x'' + y' = t, \\ y'' - x' = 1; \end{cases} \quad x(0) = 1, \quad x'(0) = -1, \quad y(0) = 1, \quad y'(0) = 0.$$

$$3. \begin{cases} x'' - y' = t, \\ y'' + x' = 0; \end{cases} \quad x(0) = -1, \quad x'(0) = 2, \quad y(0) = 1, \quad y'(0) = 0.$$

$$4. \begin{cases} x'' + y'' = 0, \\ x' + y = 1 + e^t; \end{cases} \quad x(0) = 0, \quad x'(0) = 2, \quad y(0) = 0, \quad y'(0) = -1.$$

$$5. \begin{cases} x'' + 2x' + y' = e^{-t}, \\ y'' - x' = 0; \end{cases} \quad x(0) = 0, \quad x'(0) = 1, \quad y(0) = -1, \quad y'(0) = 0.$$

$$6. \begin{cases} x'' - y = te^t, \\ x'' - x' + y'' - y = e^t + 2t; \end{cases} \quad x(0) = 0, \quad x'(0) = 1, \quad y(0) = 0, \quad y'(0) = 2.$$

$$7. \begin{cases} x'' + y' = \operatorname{sh} t - \sin t - t, \\ y'' + x' = \operatorname{ch} t - \cos t; \end{cases} \quad x(0) = 0, \quad x'(0) = 2, \quad y(0) = 1, \quad y'(0) = 0.$$

$$8. \begin{cases} x'' + x' - y' = 1, \\ x' + x - y'' = 1 + 4e^{-t}; \end{cases} \quad x(0) = 1, \quad x'(0) = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

$$9. \begin{cases} x'' - y' + y = \cos t - t, \\ y'' + x' = -2t; \end{cases} \quad x(0) = 0, \quad x'(0) = 1, \quad y(0) = 2, \quad y'(0) = -1.$$

$$10. \begin{cases} x'' - x' + y' = e^{-t} + \cos t, \\ x' - y'' - y' = 2e^t + \sin t; \end{cases} \quad x(0) = 2, \quad x'(0) = 1, \quad y(0) = 0, \quad y'(0) = 1.$$

$$11. \begin{cases} x'' + x' + y = t, \\ x' + x - y'' = 1; \end{cases} \quad x(0) = 1, \quad x'(0) = 1, \quad y(0) = 1, \quad y'(0) = 0.$$

$$12. \begin{cases} x'' - x - 2y' = t, \\ x'' - x' - y'' = 1; \end{cases} \quad x(0) = 0, \quad x'(0) = 0, \quad y(0) = 2, \quad y'(0) = 1.$$

$$13. \begin{cases} x'' + x - 2y = 2 \cos t, \\ x' - y'' = 0; \end{cases} \quad x(0) = 1, \quad x'(0) = 2, \quad y(0) = 0, \quad y'(0) = 1.$$

$$14. \begin{cases} x'' + x + 2y' = 2, \\ x' + y'' = \cos t; \end{cases} \quad x(0) = 1, \quad x'(0) = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

$$15. \begin{cases} x'' - y' = 0, \\ x - y'' = 2 \sin t; \end{cases} \quad x(0) = -1, \quad x'(0) = 1, \quad y(0) = 1, \quad y'(0) = 1.$$

Задача 2. СИСТЕМА ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ (2 балла)

Проинтегрировать систему линейных дифференциальных уравнений при заданных начальных условиях.

$$16. \begin{cases} x'' - y' = 0, \\ x' - y'' = 2 \cos t; \end{cases} \quad x(0) = 0, \quad x'(0) = 2, \quad y(0) = 2, \quad y'(0) = 0.$$

$$17. \begin{cases} x'' - x + y' = \cos t, \\ x' + y'' + y = \operatorname{ch} t + t; \end{cases} \quad x(0) = 1, \quad x'(0) = 1, \quad y(0) = 0, \quad y'(0) = 2.$$

$$18. \begin{cases} x'' + y' = 0, \\ y'' + x' = 1 - 2 \sin t; \end{cases} \quad x(0) = 1, \quad x'(0) = 1, \quad y(0) = 1, \quad y'(0) = 1.$$

$$19. \begin{cases} x'' - x + y = 1 - \frac{t^2}{2}, \\ x' + y'' = e^t; \end{cases} \quad x(0) = 1, \quad x'(0) = 2, \quad y(0) = 1, \quad y'(0) = 1.$$

$$20. \begin{cases} x'' - 2y = e^t, \\ y'' + 2x = -3e^t; \end{cases} \quad x(0) = -1, \quad x'(0) = 0, \quad y(0) = 0, \quad y'(0) = 0.$$

$$21. \begin{cases} x'' + 2y = e^{-t}, \\ x' + x - y'' + y = 0; \end{cases} \quad x(0) = 0, \quad x'(0) = 2, \quad y(0) = 1, \quad y'(0) = 0.$$

$$22. \begin{cases} x'' - x + 2y = t, \\ y'' - x' = te^{-t}; \end{cases} \quad x(0) = 0, \quad x'(0) = 1, \quad y(0) = 1, \quad y'(0) = -.$$

$$23. \begin{cases} x'' - y' = e^t, \\ y'' - y + x' = 0; \end{cases} \quad x(0) = 1, \quad x'(0) = 0, \quad y(0) = -1, \quad y'(0) = 0.$$

$$24. \begin{cases} x'' - x' + y = \sin t, \\ y'' + y - x' = e^t; \end{cases} \quad x(0) = -1, \quad x'(0) = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

$$25. \begin{cases} x'' + y' + y = e^t - t, \\ x' - x + 2y'' - y = -e^{-t}; \end{cases} \quad x(0) = 1, \quad x'(0) = 2, \quad y(0) = 0, \quad y'(0) = 0.$$

$$26. \begin{cases} x'' - y = \operatorname{sh} t - t, \\ y'' + x' = \operatorname{ch} t - 1; \end{cases} \quad x(0) = 1, \quad x'(0) = -1, \quad y(0) = 1, \quad y'(0) = 0.$$

$$27. \begin{cases} x'' + y' = \operatorname{sh} t, \\ y'' - y + x' = 2t; \end{cases} \quad x(0) = 0, \quad x'(0) = 0, \quad y(0) = 1, \quad y'(0) = -1.$$

$$28. \begin{cases} x'' + 2y' = 0, \\ x' + y'' + 2y = -4e^{2t}; \end{cases} \quad x(0) = 1, \quad x'(0) = 0, \quad y(0) = 0, \quad y'(0) = -2.$$

$$29. \begin{cases} x'' + y' + x = e^t, \\ y'' + x' = 1; \end{cases} \quad x(0) = 1, \quad x'(0) = 2, \quad y(0) = 0, \quad y'(0) = -1.$$

$$30. \begin{cases} 2y' - x'' = 2, \\ y'' + 2y + x' = 2t + 1; \end{cases} \quad x(0) = 1, \quad x'(0) = 1, \quad y(0) = 0, \quad y'(0) = -1.$$