APPROXIMATE CALCULATION OF CONVECTIVE HEAT TRANSFER NEAR HYPERSONIC AIRCRAFT SURFACE

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A new approximate method to determine the aerodynamic coefficient and the convective heat transfer coefficient is presented. A simplified mathematical model is constructed of heat exchange processes in a laminar and turbulent boundary layer, which appears near the surface of an aircraft moving with hypersonic velocity. This mathematical model makes possible the calculation of the enhanced heat and mass transfer on the surface of a hypersonic aircraft of a complex geometric shape, including at low temperatures.

KEY WORDS: *heat transfer enhancement, hypersonic jet, mathematical model, boundary layer, computational aerodynamics*

1. INTRODUCTION

Hypersonic tunnels and planes (HP) have many special design problems. Two specific physical problems are 1. friction coefficient effects on movement at hypersonic speed and also viscous tangent tension on the surface of a streamline body at various modes of a current; 2. effects of a convective stream on thermo-physical characteristics, and determination of external thermal loads of a design of hypersonic aircraft. Various methods and approaches have been applied for hypersonic flows and aerodynamic applications (Terekhov et al., 1997; Simon and Jiang, 2009; Ghosh and Goldstein, 2014; Kuzenov and Ryzhkov, 2017, 2018).

Both specified problems are associated with thermal and dynamic boundary layers that appear on the external surface of a hypersonic aircraft moving in a continuous environment. In an aircraft moving at a hypersonic speed, the speed gradients across the boundary layer increase noticeably (together with frictional forces) if they are compared to a vehicle at supersonic speed. In a narrow (compared with the characteristic size of a streamline body) hypersonic boundary layer, the friction forces lead to intensive energy release (i.e., a boundary layer can be considered as a narrow space area adjoining the surface of a streamline body in which there is an intensive allocation of heat at the expense of energy dissipation). These processes of dissipation are followed by a strong change in all physical (density, pressure, temperature, viscosity, heat conductivity, etc.) and dynamic properties of gas and also thermal streams directed to the surface of the aircraft. These processes require analytical and numerical analyses.

The aim of the current study is the development of a method for estimating convective heat streams and the friction coefficient by engineering methods for bodies of complex shape. The main part of such research is based on the mathematical model of the boundary layer (the effective length method) (Abramovich, 1973; Avduevsky and Poleshaev, 1991; Avuduevskii et al., 1992; Alekseev and Zhurin, 2006; Lunev, 2017). In this case, laminar and turbulent boundary layer models are used (laminar-turbulent transition issues were not considered). It is important to note that the calculation of the heat stream must be preceded by calculation of the external flow. Determination of external thermal loads is an important step in determining the temperature conditions of the HP design. At present, there are several approaches to the calculation of convective heat transfer near the surface of hypersonic vehicle.

The first method is numerical integration of the Navier-Stokes equations. In this case, the problems of calculating convective heat transfer on bodies of complex shape and in the separation zones are also solved (Bauer, 1991; Voinov, 1997; Surzhikov, 2011; Zheleznyakova and Surzhikov, 2014). The results of the calculations are in good agreement with the results of both natural experiments and those obtained on the shock tubes. This direction of development of computational hydro and gas dynamics is the most promising. The solution of such tasks requires a large cost of computer time. The results of calculations can strongly depend on the structure of the computational grid, the size of the calculated area, the input parameters, and the particulars of the calculation algorithm. Thus, the Navier-Stokes equations for the primary evaluation of heat fluxes are problematic enough.

The second method of calculating heat streams is to calculate the structure of the boundary layer on the basis of the Prandtl equations (Lunev, 2017). For large Reynolds numbers ($\mathrm{Re} > 10^4$–10^5) and in the absence of flow separation zones, the flow around the bodies can be divided into two areas: an inviscid, the main on volume, in which the Euler equations are described, and the wall boundary layer (Abramovich, 1973; Avduevsky and Poleshaev, 1991; Avuduevsky et al., 1992; Alekseev and Zhurin, 2006; Lunev, 2017). The following estimate of the thickness of the boundary layer is: $\delta \approx 5 \cdot \sqrt{\mu\ell/(\rho U)}$, where μ is the dynamic viscosity of the gas, ℓ is the characteristic length of the streamlined object, ρ is the characteristic gas density, and U is the characteristic speed of the incoming flow. From this it can be seen that the thickness of the boundary layer $\delta \approx 5 \cdot \ell/\mathrm{Re}^{0,5}$ with $\mathrm{Re} = \rho U\ell/\mu \approx 10^5$ is a small part of the characteristic size of the streamlined body. Thus, in the first approximation, it can be neglected and an inviscid flow field can be obtained within the framework of the Euler equations system. The gas dynamic parameters on the surface of the body can be also considered as parameters at the outer boundary of the boundary layer. Methods for calculating the planar and three-dimensional boundary layers were developed by Lunev (2017). The task is divided into several stages of calculation, setting of boundary conditions, and conjugation of computational grids. However, such a method of estimating heat fluxes is also quite time consuming.

A third estimation of heat flows assumes the determination of the main characteristics of the boundary layer without determining its structure by the methods of local similarity (Abramovich, 1973; Avduevsky and Poleshaev, 1991; Bauer, 1991; Avuduevskii et al., 1992; Voinov, 1997; Alekseev and Zhurin, 2006; Surzhikov, 2011). For the boundary layer on the surface of a complex shape, an analogy is created with the body of the simplest form, for example, a plate or a cone. At the same time, geometric parameters of simple bodies are selected for each section of a complex surface, the laws of development of the boundary layer on which are known. To determine the heat flux in this way, it is necessary to know the distribution of gas dynamic parameters at the outer boundary of the boundary layer. Note that this approach is applicable only where the thin boundary layer model works. In tear-off zones, this method can give a qualitatively incorrect result, because the separation has a viscous nature (Abramovich, 1973; Lunev, 2017). Previous researchers (Voinov, 1997; Surzhikov, 2011; Zheleznyakova and Surzhikov, 2014; Lunev, 2017) describe in detail the methods and results of estimating convective heat fluxes by the methods of local similarity.

2. MATHEMATICAL ASPECTS OF THE APPROXIMATE METHOD OF CALCULATING HEAT FLOW TO THE SURFACE

The methodology for calculating the aerodynamic heating of a complex geometric shape can be represented (from the computational point of view) by several steps:

1. Search for a deceleration point (critical point) on the surface of a HP of a complex geometric shape for arbitrary flow conditions;
2. Determination of the current line for any point on the surface of a HP (specified by an unstructured grid), both in the direction of the flow and against the direction of flow of the gaseous medium. Formulation of an algorithm for localizing the position of a point on the streamline with respect to a 2D finite-element decomposition of the surface of a HP;
3. Finding metric coefficients on the basis of ratio of differential geometry (Shikin and Berezin, 2007);
4. Calculation of the direction of the flow (along the current line found) with the required degree of accuracy for the specific integrals;
5. Separate from the main method for calculating the convective heat flux near the critical point.

Before discussing the points formulated, let us recall once again that in order to determine the convective heat fluxes on the basis of the approximate physical and mathematical model of aerodynamic model, it is necessary first to calculate (on the basis of the three-dimensional Euler equations) the distribution of the gas dynamic parameters ($\rho, \overrightarrow{V}, P$) at the outer boundary of the boundary layer near the surface of a streamlined body.

An important stage in the numerical modeling of convective heat transfer on the surface of a HP is the generation of a calculated surface grid, which is performed using information on the geometric 3D model of the surface of a HP. At the same time, the discretization of the composite surface consisting of nonuniform rational B-splines segments containing discontinuities and complex boundaries is a significant complication. This problem is solved by constructing a triangular grid on the surface of a HP using algorithms of two-dimensional triangulation in combination with parametric mappings.

Thus, in the three-dimensional version of the approximate physical and mathematical model, the surface of a streamlined body is divided into a set of triangles, or facets. The vertices of the facets are called nodes. All facets are uniquely conjugated to each other through the edges of the facets. The absence of defects on the triangulated surface is an important condition for the correct operation of the calculation algorithm. The use of a superficial unstructured grid for estimating heat fluxes on the basis of an approximate physical and mathematical model makes it possible to investigate the aerodynamic heating of a HP of complex geometric shapes. This technology (in principle) makes it possible not to introduce a general curvilinear coordinate system (s, β, n) associated with the geometric shape of the streamlined body, and to set the gas dynamic parameters in its own, independent coordinate system $(u,\ v, s)$ only for a single arbitrary facet.

The search for the deceleration point on the surface of a HP in this technique is based on finding extremes of static pressure and uses the method of minimizing the function of several variables: a simpler or a minimization method in given directions.

The definition of the streamline for an arbitrary point on the surface of a HP is based on the numerical (using the explicit Euler method) solution of the vector ordinary differential equation:

$$\frac{d\overrightarrow{X}}{dt} = \overrightarrow{V}, \quad \overrightarrow{X}(t = t_s) = \overrightarrow{X}_s, \quad \overrightarrow{X}(t) = (x, y, z)^T, \quad \overrightarrow{V}(t) = (u, v, w)^T. \tag{1}$$

We note that in order to maintain the accuracy and correct operation of the algorithm, the step $\Delta\overrightarrow{X}$ with which the current line is restored should be less than the characteristic size of the facet.

To find the solution of Eq. (1) in each node of the calculated surface unstructured grid, it is necessary to specify the gas dynamic parameters of the flow: P is the pressure, T is the temperature, and u, v, w are the components of the velocity vector $\overrightarrow{V}(t)$. Assuming that the values of the gas dynamic parameters of the flow in the center of the tetrahedron are known, the values of these parameters at the grid nodes can be found (Grigoryev et al., 2002) using conservative linear interpolation:

$$F_P \approx \frac{\sum F_i V_i}{\sum V_i}, \tag{2}$$

where F_i is the value of the function at the center of the i-th tetrahedron, and V_i is the volume. The summation is carried out over all tetrahedral containing the vertex P.

Inside the facet, all the gas dynamic parameters are determined by linear interpolation using linear basis functions

$$N_k^i(r_j) : F(r_j) = \sum_{k=1}^{d+1} N_k^i(r_j)\, F\left(r_k^i\right),$$

here $F(r_j)$ are the values of the interpolated parameter given at the vertices of the facet.

For two space variables in a local coordinate system (u, v, s) for an arbitrary 2D finite element (facet) C_i with local numbering of nodes (vertices) $k = \overline{1, 3}$, the linear basis functions $N_k^i(r_j)$ are defined by the ratio:

$$N_k^i(r) = \alpha_k^i + \beta_k^i x^* + \gamma_k^i y^*, N_k^i\left(r_m^i\right) = \delta_{km}. \tag{3}$$

Given a partition (Grigoryev et al., 2002) (triangulation) of the 2D surface of a streamlined body into a set of triangles, we have three (equal to the number of nodes in the facet) of the system of linear algebraic equations on each facet:

$$\begin{pmatrix} 1 & x_1^{*,i} & y_1^{*,i} \\ 1 & x_2^{*,i} & y_2^{*,i} \\ 1 & x_3^{*,i} & y_3^{*,i} \end{pmatrix} \begin{pmatrix} \alpha_k^i \\ \beta_k^i \\ \gamma_k^i \end{pmatrix} = \begin{pmatrix} \delta_{1,k} \\ \delta_{2,k} \\ \delta_{3,k} \end{pmatrix}. \tag{4}$$

The coefficients of a given linear form are computed as solutions of systems of linear algebraic equations whose determinant Δ_i has the form:

$$\Delta_i = \begin{vmatrix} 1 & x_1^{*,i} & y_1^{*,i} \\ 1 & x_2^{*,i} & y_2^{*,i} \\ 1 & x_3^{*,i} & y_3^{*,i} \end{vmatrix} = \left(x_2^{*,i} - x_1^{*,i}\right)\left(y_3^{*,i} - y_1^{*,i}\right) - \left(x_3^{*,i} - x_1^{*,i}\right)\left(y_2^{*,i} - y_1^{*,i}\right), \quad \left(k = \overline{1,3}\right). \tag{5}$$

For facets having a finite area, this system is uniquely solvable, and its solution can be found by the Cramer method or by a suitable numerical method. It is necessary to formulate an algorithm for localizing the position of an arbitrary point located on the streamline with respect to the 2D finite element of the HP surface. Effective procedures for the localization of the streamline on an unstructured grid consisting of triangular elements in the case of two spatial dimensions and tetrahedral in a three-dimensional grid are given by Grigoryev et al. (2002). These procedures are based on the idea of sequentially searching for the position (for the moment of time $t_n + \Delta t$) of the point of the current line calculated in (1) for the nearest neighboring cells after its displacement from the original facet at the next time step $t_n + \Delta t$. To do this, we use local basis functions $N_k^i\,(r_j)$, which perform the role of local coordinates here. It is assumed (Grigoryev et al., 2002) that for a given 2D finite-element partition of the HP surface, each facet introduces its own, local coordinate system (u, v, s), in which for each of the 2D elements, the coordinates of the nodes (vertices) with the use of systems (4) local bases N_k^i.

Let us formulate the localization criterion in a facet belonging to the 2D finite-element partition of the HP surface, an arbitrary point of the streamline (Grigoryev et al., 2002). We put the center of the point on the streamline at the time moment t_n at a point $r^n \in C_i$, where C_i denotes an arbitrary facet on the surface of the body being streamlined. In accordance with the definition of linear basis functions for any $r \in C_i$ for all $k = 1, ..., d+1$ basis functions $0 \leqslant N_k^i(r) \leqslant 1$. Hence, if $n+1$ for the moment of time $t_{n+1} = t_n + \Delta t$ the relations $\min N_k^i(r^{n+1}) \geqslant 0$, and $\max N_k^i(r^{n+1}) \leqslant 1$, $k = \overline{1, d+1}$, are satisfied, then the point $r^{n+1} \in C_i$ is located in the facet C_i. If at least for one of the nodes (vertices) $\min N_k^i(r^{n+1}) < 0$, then the point r^{n+1} is not located facet $r^{n+1} \notin C_i$. Then in the case of two spatial dimensions, if $r^{n+1} \notin C_i$, the search should continue in the neighboring element, opposite to the k-th vertex of the element C_i for which

$$N_k^i\left(r^{n+1}\right) = \min_{\ell} N_\ell^i\left(r^{n+1}\right) < 0.$$

For the transition to our own calculation (connected with 2D finite element-facet), local independent (from the initial Cartesian coordinate system) coordinate system (u, v, s) for some arbitrary facet, we can use matrices of the third order $[T]$, $[R_x]$, $[R_y]$, Euler angles θ, ψ, and the transfer of the initial Cartesian coordinate system (x, y, z) to the “center of mass” of the facet (x_m, y_m, z_m).

To do this, we perform the following sequence of basic transformations (the complete transformation is obtained by the composition of the base transformations (Shikin and Boreskov, 1996):

- transfer of the vector V to the origin (transformation $[T]$);
- rotation of the axis Z by an angle ψ around the axis X (transformation $[R_x]$);

- rotate the transformed coordinate system around the axis Y by an angle θ (transformation $[R_y]$).

The transfer of the vector V to the origin of the local coordinate system (u, v, s) is described by:

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x_m & -y_m & -z_m & 1 \end{bmatrix}, \tag{6}$$

where the symbols x_m, y_m, z_m indicate the coordinates [in the initial Cartesian coordinate system (x, y, z)] of the geometric "center of mass" of the facet.

Rotate the axis z by an angle ψ about the axis x of the initial Cartesian coordinate system (x, y, z):

$$\cos(\psi) = \frac{n}{d}, \quad \sin(\psi) = \frac{m}{d}, \quad d = \sqrt{n^2 + m^2}, \tag{7}$$

$$[R_x] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\psi) & \sin(\psi) & 0 \\ 0 & -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Rotate the transformed coordinate system around the axis y by an angle θ:

$$\cos(\theta) = d, \quad \sin(\theta) = \ell, \quad [R_y] = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

The resulting linear transformation has the following form:

$$[u \quad v \quad s \quad 1] = [x \quad y \quad z \quad 1]\,[T]\,[R_x]\,[R_y], \tag{9}$$

where (u, v, s) denotes the values of the coordinates of a certain point in the facet in a local coordinate system associated with an arbitrary facet; (x, y, z) is the value of the coordinates of the same point in the original Cartesian coordinate system.

We note that the rotation (at angles ψ and θ) occurs around a certain straight line with a directing vector $V(\ell, m, n)$, where $\ell^2 + m^2 + n^2 = 1$.

The calculation of the integrals requires finding the metric coefficients (the local mean radius of curvature) that enter the integrand. We recall the basic propositions used when finding the local mean radius of curvature of a smooth curvilinear surface (Shikin and Berezin, 2007).

Suppose that a smooth or regular surface S is locally defined in a three-dimensional Euclidean space. Such a task is possible if a vector relation is given:

$$\overrightarrow{r} = \overrightarrow{r}(u, v), \quad (u, v) \in U \times V, \tag{10}$$

where U, V are the intervals of variation of the variables u, v. We also assume that there exist continuous derivatives $\overrightarrow{r_u} = \partial\overrightarrow{r}(u, v)/\partial u$, $\overrightarrow{r_v} = \partial\overrightarrow{r}(u, v)/\partial v$ for intervals U, V which satisfy an additional condition of the form: $\overrightarrow{r_u} \times \overrightarrow{r_v} \neq 0$. Then on the surface S one can define a curve

$\overrightarrow{r} = \overrightarrow{r}(u(t), v(t)), (u,v) \in U \times V$, introduce its differential $d\overrightarrow{r} = \overrightarrow{r_u}du + \overrightarrow{r_v}dv$, and also the vector of the unit normal

$$\overrightarrow{N} = \frac{\overrightarrow{r_u} \times \overrightarrow{r_v}}{|\overrightarrow{r_u} \times \overrightarrow{r_v}|}.$$

In this case, the square of the differential of the arc length ds^2 determines the first fundamental quadratic form of the surface S (Shikin and Berezin, 2007):

$$ds^2 = |d\boldsymbol{r}|^2 = E(u,v)du^2 + 2F(u,v)dudv + G(u,v)dv^2, \tag{11}$$

where

$$E(u,v) = \boldsymbol{r}_u \cdot \boldsymbol{r}_u = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2,$$

$$F(u,v) = \boldsymbol{r}_u \cdot \boldsymbol{r}_v = \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v},$$

$$G(u,v) = \boldsymbol{r}_v \cdot \boldsymbol{r}_v = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2.$$

We define the differential of the unit normal by the expression $d\overrightarrow{N} = \overrightarrow{N_u}du + \overrightarrow{N_v}dv$ and introduce the following notation:

$$L(u,v) = -\boldsymbol{r}_u \cdot \boldsymbol{N}_u = \frac{[\boldsymbol{r}_{uu}, \boldsymbol{r}_u, \boldsymbol{r}_v]}{\sqrt{EG - F^2}},$$

$$M(u,v) = -\boldsymbol{r}_u \cdot \boldsymbol{N}_v = \frac{[\boldsymbol{r}_{uv}, \boldsymbol{r}_u, \boldsymbol{r}_v]}{\sqrt{EG - F^2}},$$

$$N(u,v) = -\boldsymbol{r}_v \cdot \boldsymbol{N}_v = \frac{[\boldsymbol{r}_{vv}, \boldsymbol{r}_u, \boldsymbol{r}_v]}{\sqrt{EG - F^2}}.$$

Then the average local radius of curvature R_{ef} can be found as follows:

$$K = \frac{1}{2}\frac{EN - 2FM + GL}{EG - F^2}, \quad R_{ef} = \frac{1}{K}. \tag{12}$$

We note that in the case when the surface S is given in a three-dimensional Euclidean space with a metric $du^2 + dv^2 + ds^2$ as the graph of a differentiable function $s = f(u,v)$, the relation (12) acquires the following symmetric form (Pogorelov and Noordhoff, 1974):

$$K = \frac{1}{2}\frac{\left(1 + f_u^2\right)f_{vv} - 2f_u f_v f_{uv} + \left(1 + f_v^2\right)f_{uu}}{\left(1 + f_u^2 + f_v^2\right)^{3/2}}, \quad R_{ef} = \frac{1}{K}. \tag{13}$$

The partial derivatives $f_u, f_v, f_{uv}, f_{uu}, f_{vv}$ entering into the relation (13) can be found by the method of least squares for the function $f(u,v)$ containing the quadratic terms and approximating the shape of the surface S at the point (u,v):

$$\begin{aligned} f(u,v) \approx f(u_j, v_j) + f_u(u - u_j) + f_v(v - v_j) + f_{uu}\frac{(u - u_j)^2}{2} \\ + f_{uv}(u - u_j)(v - v_j) + f_{vv}\frac{(v - v_j)^2}{2}. \end{aligned} \tag{14}$$

Applying the method of least squares to the expression (14), we can formulate the following system of linear algebraic equations for finding the derivatives $\overrightarrow{c} = (f_u, f_v, f_{uu}, f_{uv}, f_{vv})^T$:

$$P\overrightarrow{c} = \overrightarrow{Q}, P_{kj} = \sum_{i=1}^{m} \varphi_k(t_i)\varphi_j(t_i), Q_k = \sum_{i=1}^{m} \varphi_k(t_i)(f_i - f_j), \tag{15}$$

where

$$\varphi_j = \{(u-u_j), (v-v_j), (u-u_j)^2, (u-u_j)(v-v_j), (v-v_j)^2\},$$

$$\overrightarrow{c} = (f_u, f_v, f_{uu}, f_{uv}, f_{vv})^T.$$

The values of integrals

$$\int_0^S \rho^* \mu^* \left|\overrightarrow{V}\right| r^2 ds \quad \text{(laminar flow)},$$

and

$$\int_0^S \rho^* (\mu^*)^m \left|\overrightarrow{V}\right| r^{c_3} ds \quad \text{(turbulent flow)},$$

must be obtained at each point (except the deceleration point) of the surface of the HP. Following the conclusions of Churakov et al., (2009), we can assume that the most preferable in this situation is the adaptive Newton–Cotes method, which is a family of quadrature formulas obtained by integrating interpolation polynomials constructed from equidistant nodes, and as a method of interpolation of the integrand, we can use cubic spline interpolation (De Boor, 1978). It is possible (in the laminar case) to approximately determine the value of the integral

$$\int_0^S \rho^* \mu^* V r^2 ds,$$

near the critical point. In this case, the approach is based on the following assumptions (Zoby et al., 1981; Hamilton et al., 2006, 2009): first, the values ρ^* and μ^* do not vary significantly along the streamline and can be replaced with their values at a critical point with good approximation $\rho^*\mu^* = (\rho^*\mu^*)_s$; second, the magnitude of the velocity modulus V (recall, at the critical point $V_s = 0$) and the enthalpies h can be represented by a linear dependence: $V \approx (dV/ds)_s\, s$, $h \approx (dh/ds)_s\, s$. Thus, the parameter g_L near the critical point $(s \approx S_\varepsilon)$ can be calculated in an approximate way:

$$\begin{aligned}\int_0^{S_\varepsilon} \rho^*\mu^* V h^2 ds &= \int_0^{S_\varepsilon} (\rho^*\mu^*)_s \left(\frac{dV}{ds}\right)_s \left(\frac{dh}{ds}\right)_s^2 s^3 ds \\ &\approx (\rho^*\mu^*)_s V_\varepsilon h_\varepsilon^2 \frac{1}{s_\varepsilon^3} \int_0^{s_\varepsilon} s^3 ds = (\rho^*\mu^*)_s V_\varepsilon h_\varepsilon^2 \frac{s_\varepsilon}{4}.\end{aligned} \tag{16}$$

The above algorithm for calculating the heat flux near the critical point has a singularity of the type $0/0$. For this reason, the simplified formula of Fay and Riddell (1958) and Martin (1966) is used near the deceleration point (critical point):

$$q_w = 2.3 \cdot 10^7 \sqrt{\frac{1}{R_n}} \left(\frac{V_\infty}{10^6}\right)^{3.15} \sqrt{\frac{\rho_\infty}{\rho_0}}, \quad W/cm^2, \tag{17}$$

where $\rho_0 = 1.23 \cdot 10^{-3}$ g/cm^3. The direct expression describes the transfer of energy through a stationary (near critical point) dissociating gas with temperature ∇T and concentration gradients ∇c_i (Surzhikov, 2013):

$$q_{w,i} = -\lambda_i \nabla T - \rho D_i h_i^0 \nabla c_i, \tag{18}$$

where λ_i is the coefficient of thermal conductivity of the$i$-th component of the mixture, ρ, T density and temperature of the solid of the medium at the boundary layer boundary, h_i^0 is the specific dissociation energy per unit mass of the i-th component of the mixture, D_i is the diffusion coefficient of the i-th component of the mixture, and c_i is the mass fraction of the i-th component of the mixture.

The following is one of the methods for calculating the aerodynamic coefficients of forces operating on a HP. For a single element dS of the surface S, the force of pressure $d\overrightarrow{F_p}$ acting along the normal $\overrightarrow{n}$ to the surface S. This force can be specified with the help of the expression: $d\overrightarrow{F_p} = (P - P_\infty)\overrightarrow{n} dS$. Then the aerodynamic pressure coefficient $\overrightarrow{C_p}$ can be found using formula:

$$\overrightarrow{C_p} = \frac{\overrightarrow{F_p}/S_{mid}}{\rho_\infty V_\infty^2/2} = \left(\frac{1}{S_{mid}} \iint\limits_S (P - P_\infty)\overrightarrow{n} dS\right) / \left(\rho_\infty V_\infty^2/2\right),$$

where S_{mid} is the middel area, $\rho_\infty, P_\infty, V_\infty$ are the density, pressure and speed in the unperturbed, running on HP gas stream.

The values of the components of the aerodynamic coefficient $\overrightarrow{C_p}$ in the axial C_{px} and transverse C_{py} directions are obtained by projecting the normal $\overrightarrow{n}$ to the regular surface S on the directing vectors of basis vectors e_x, e_y of a Cartesian coordinate system (x, y, z):

$$C_{px} = \left(\frac{1}{S_{mid}} \iint\limits_S (P - P_\infty)(\overrightarrow{n} e_x)\, dS\right) / \left(\rho_\infty V_\infty^2/2\right),$$

$$C_{py} = \left(\frac{1}{S_{mid}} \iint\limits_S (P - P_\infty)(\overrightarrow{n} e_y)\, dS\right) / \left(\rho_\infty V_\infty^2/2\right),$$

where e_x, e_y are the directing vectors of basis vectors of the Cartesian coordinate system.

In the viscous flow past a HP (and the corresponding viscous calculation), a friction force $d\overrightarrow{F_f}$ acts on the unit element dS of the surface S, directed along the tangent to the HP surface collinearly to the velocity vector. This force can be found as follows: $d\overrightarrow{F_f} = \tau_w \overrightarrow{V} dS$, where τ_w is the frictional stress determined by the viscous forces, $\overrightarrow{V}$ a unit vector tangential to the surface S. Then the coefficients of surface friction C_{fx}, C_{fy} are written as follows:

$$C_{fx} = \left(\frac{1}{S_{mid}} \iint\limits_S \left(\overrightarrow{V} e_x\right) \tau_w dS\right) / \left(\rho_\infty V_\infty^2/2\right),$$

$$C_{fy} = \left(\frac{1}{S_{mid}} \iint\limits_{S} \left(\overrightarrow{V} e_y \right) \tau_w dS \right) / \left(\rho_\infty V_\infty^2 / 2 \right).$$

The total aerodynamic coefficients C_x, C_y can be represented as follows:

$$C_x = C_{px} + C_{fx}, \quad C_y = C_{py} + C_{fy}.$$

When a HP moves at an angle α of attack, the aerodynamic coefficients are determined by the formulas:

$$C_{xa} = C_x \cos\alpha + C_y \sin\alpha, \quad C_{ya} = -C_x \sin\alpha + C_y \cos\alpha,$$

where C_{xa} is the coefficient of aerodynamic resistance (aerodynamic drag coefficient), C_{ya} is the aerodynamic lift coefficient.

3. CALCULATION RESULTS

For the purposes of 2D validation and verification of the mathematical model of heat transfer in hypersonic flow around blunt axisymmetric bodies, the calculated and experimental data given in Surzhikov (2013) were used. These calculations were performed for Mach numbers M $= 6$ and altitudes $h = 22$ km and $h = 37$ km. Initial data for calculations of flow around a spherically blunted cylinder: $P_\infty = 0.23\ 10^4$ Tor, $\rho_\infty = 0.178\ 10^{-5}$ g/cm^3, $V_\infty = 4.167\ 10^5$ cm/c, $T_\infty =$ 450 K. Initial testing calculations were performed using unstructured grids and the NERAD 3D computational code.

Direct (full-scale) flight experiments using advanced aircraft (flight demonstrators) are the most complex and costly way to obtain experimental results. In this case, full-scale experiments are required for the development of:

1. hypersonic flight-experimental base on the basis of an aircraft command-and-measurement station, which is designed to test hypersonic technology on demonstrators;

2. systems for pre-acceleration and demystification for hypersonic test modes (based on solid-fuel accelerators);

3. ground infrastructure for the preparation of test facilities, testing and preflight testing of demonstrators and test objects.

Thus, it is advisable to apply the method of experimental study of enhanced heat and mass transfer (at low temperatures) on a reduced (geometrically similar) model of a prospective aircraft instead of a full-scale study of the aerothermogas-dynamics of the flow near the surface of advanced aircraft. Here we note that a decrease in the temperature to $T \approx 60$ K leads to a decrease in the speed of sound $C \sim \sqrt{T}$ and a noticeable increase in the Mach number M $= V_\infty$/C for moderate values of the velocity V_∞ that runs through the undisturbed flow. However, it should be noted that in this case some features of the flow characteristic of direct (natural) flight experiments with the accuracy necessary for practice can be reproduced only in an approximate way.

In 3D testing, the following air flow parameters flowing on the HP were used: Pressure in the flowing stream: $P = 1120$ Pa; Speed in the incoming flow: $V = 945$ m/s; Temperature in the flowing stream: $T = 62.1$ K; Height from the surface: $H = 25$ km; In the performed calculations, the angle of incidence (from 0 to 20 degrees) of the incident air flow (angle of attack) also varied.

Figures 1–3 show the individual calculation results using an approximate mathematical model of heat exchange processes for a hypersonic aircraft.

From the distribution of the Reynolds number along the surface of the aircraft (Fig. 1), it follows that the transition to a turbulent flow regime is observed almost immediately (the laminar flow region exists only near the critical front points) behind the head of the apparatus. It can also be noted that the temperature distribution along the surface of a hypersonic aircraft (Fig. 3) has a maximum (it is also observed for static pressure) at the deceleration point ($T \approx 500$ K). In this case, the bulk of the gas near the surface has a lowered temperature at the level $T \approx 150$ K.

The values of aerodynamic coefficients in the axial $C_{px} = 0.1055$ and transverse $C_{py} = 0.3789$ directions were also calculated using the method described above.

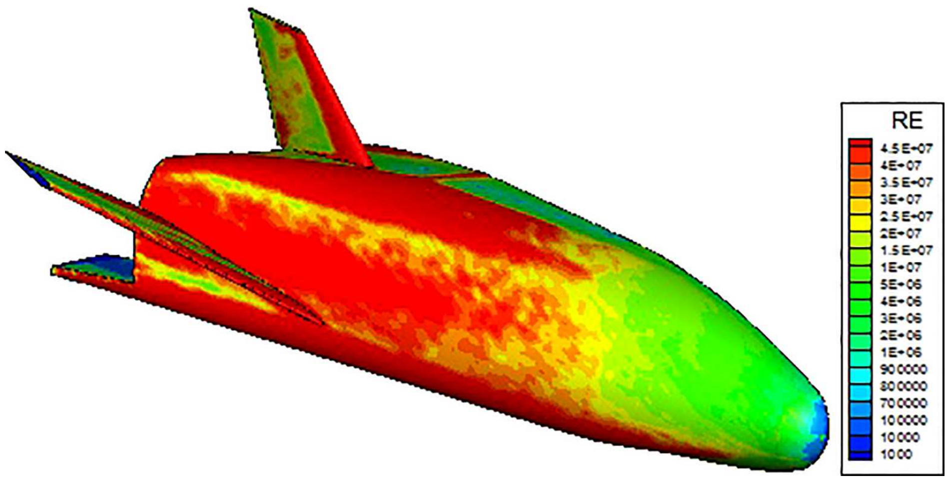

FIG. 1: Distribution of the Reynolds number along the surface of an aircraft

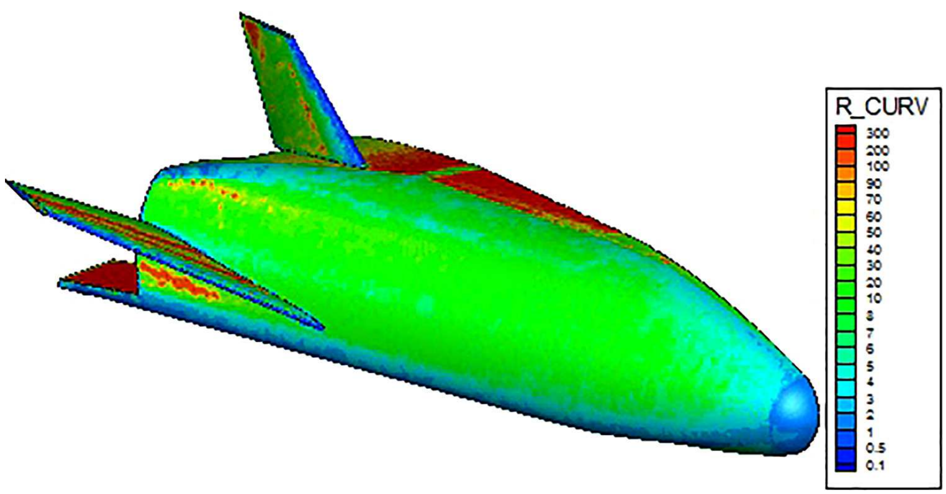

FIG. 2: Distribution of the mean radius of curvature R_{ef} [cm] of the surface of a hypersonic aircraft

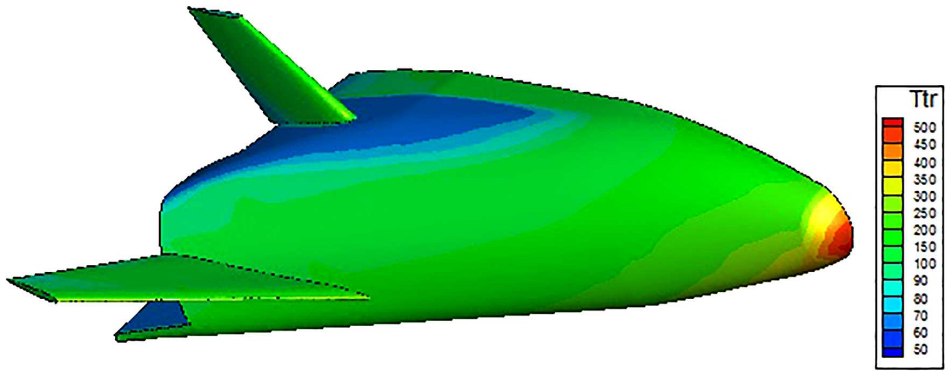

FIG. 3: Temperature distribution [K] along the surface of a hypersonic aircraft

4. CONCLUSION

This article presents results obtained in a study of the convective heat transfer on the surface of an aircraft moving at hypersonic speed. An approximate mathematical model of heat exchange processes in the laminar and turbulent boundary layer (Sivykh, 2000) is developed by one of the authors (VVK). In the developed technique, a surface triangular unstructured computational grid is used to estimate the convective heat fluxes. This makes possible the study of aerodynamic heating (convective heat transfer) on a complex geometric shape. The developed computing technology does not introduce a general curvilinear coordinate system for describing the geometry of a streamlined body and gas dynamic parameters on its surface, which makes it possible to obtain the values of the thermal parameters in each cell separately in its own independent coordinate system. The developed mathematical model can be used to calculate the enhanced heat and mass transfer on the surface of a hypersonic aircraft of a complex geometric shape, including at low temperatures (near 60 K).

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