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Developing a Procedure for Calculating Physical Processes in Combined Schemes of Plasma Magneto–Inertial Confinement

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Abstract—A combined scheme of hot plasma confinement is proposed, and laser and plasma-based methods for generating a megagauss field during the implosion of a magnetized target are described that allow the development of new high-density plasma sources for materials science experiments and advanced areas of power engineering. A procedure for numerical calculation of the physical processes involved in the target plasma in an external magnetic field is presented.

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INTRODUCTION

Experimental and theoretical investigations of the plasma magneto–inertial confinement in combination with the external magnetic field are developing rapidly [1–9]. Being in a strong frozen-in plasma magnetic field, almost all outgoing high-energy charged particles experience several acts of nuclear scattering in a high-density plasma and are contained by the compressed magnetic field. As a result, these high-energy ions expend their energy to heat the adjoining plasma areas, which is a necessary conditions for the propagation of a combustion wave.

The aim of this work was to study magnetized target compression in systems with different powerful, efficient, and safe sources of plasma heating (drivers): laser beams and ultrasonic plasma jets [10–18].

STATEMENT OF THE PROBLEM

A system of magneto–inertial hot plasma confinement is an impulse device in which a cylindrical or spherical target placed in a seed (external) magnetic field is compressed (along with the seed magnetic flux) by powerful laser beams or different types of envelopes (including gas, liquid, and metal hammers), by plasma liners formed by the merging of high-speed plasma jets, and so on.

At the stage of the initial study of the main physical patterns, it is advisable to develop and use simplified linear mathematical models. Such systems can describe physical processes in the laser or plasma drivers, and in the compressed target (including the compression of the initial magnetic flux).

A mathematical model uses multicomponent, one-dimensional equations of plasma dynamics, the equation of normal broadband radiation transfer, the equation of magnetic induction, the transfer of laser radiation in a geometric optics approximation, and methods for calculating state of matter and laser radiation absorption factors. The model determines the conditions for the initiation and behavior of a self-sustaining fusion reaction.

The system of equations for plasma dynamics is written as

$$\frac{\partial(\rho c_i)}{\partial t} + \mathrm{Div}\left(\rho c_i \vec{V}\right) = F_{\rho c}, \quad F_{\rho c} = -\frac{\rho c_i u(\nu - 1)}{r}$$

$$+ \mathrm{Div}\left(\rho q_i^D\right) + \left(\frac{\partial \rho c_i}{\partial t}\right)_x, \quad \frac{\partial \rho}{\partial t} + \mathrm{Div}(\rho u) = F_\rho,$$

$$F_\rho = -\rho u \frac{(\nu - 1)}{r}, \quad \frac{\partial(\rho u)}{\partial t}$$

$$+ \mathrm{Div}\left(\rho u^2 + P^\Sigma\right) = F_{\rho u} + f_r,$$

$$F_{\rho u} = -\rho u^2 \frac{(\nu - 1)}{r}, \quad f_r = \frac{1}{c}\left[\vec{j} \times \overline{H}\right]_r,$$

$$\frac{\partial(\rho E)}{\partial t} + \mathrm{Div}\left(\rho E u + P^\Sigma u + q^\Sigma\right)$$

$$= F_E + q_r + D_x + Q_{Fus}^e,$$

$$F_E = -\left(\rho E u + P^\Sigma u\right)\frac{(\nu - 1)}{r}, \quad P^\Sigma = P_e + P_i, \qquad (1)$$

$$\frac{\partial \rho e_e}{\partial t} + \mathrm{Div}(\rho e_e u + P_e u + q + q_e)$$

$$= F_e + q_r - Q_{ei} + Q_{Fus}^e,$$

$$F_e = -(\rho e_e u + P_e u)\frac{(\nu - 1)}{r},$$

$$\frac{\partial \rho e_i}{\partial t} + \mathrm{Div}(\rho e_i u + P u + q_i) = F_i + Q_{ei}, \quad F_i$$

$$= -(\rho e_i u + P_i u)\frac{(\nu - 1)}{r}, \quad D_x = \sum_i h_i \mathrm{Div}\left(\rho q_i^D\right),$$

$$\mathrm{Div}\left(\rho q_i^D\right) = \frac{1}{J}\frac{\partial J\left(\rho q_i^D\right)}{\partial \xi}, \quad J = r^{(\nu - 1)},$$

where index $\nu = (1,2,3)$ satisfies the condition of flat, axial, and spherical symmetry; t is time; r is a spatial coordinate; ρ is density; u is the plasma velocity along the r coordinate; $P = P(\rho,\varepsilon)$ is the static pressure; ε is the specific internal energy; $E = \left(\varepsilon + u^2/2\right)$ is the strength of the gas flow; $\vec{F} = (F_\rho, F_{\rho u}, F_E)$ is the vector of sources; j is the current density; q, q_ν are the total and spectral radiation flows in the direction of axis r; T_e, T_i are the temperatures of the plasma electrons and ions; ν is the number of the frequency group; χ_ν is the spectral absorption coefficient, f_r is the electromagnetic force; q_r is the influx of the energy of the electromagnetic field; $q_e = -\lambda_\perp^e \operatorname{grad} T_e$ and $q_i = -\lambda_\perp^i \operatorname{grad} T_i$, $q_i^D = -D_\perp^i \operatorname{grad}(c_i)$ are the thermal and diffusion flows that arise due to the processes of thermal productivity and diffusion; $\lambda_\perp^e$ and $\lambda_\perp^i$ are the thermal conductivity coefficients of electrons and ions; and $D_\perp^i$ is the coefficient of diffusion ions.

The transfer of the broadband radiation in one-dimensional problems can be considered using a multi-group diffusion approximation with the equations [19]

$$\frac{1}{r}\frac{d\left(r^n q_\nu\right)}{dr} + \chi_\nu c U_\nu = \chi_\nu 4\sigma T^4, \quad \frac{c}{3}\frac{dU_\nu}{dr} + \chi_\nu q_\nu = 0,$$

$$q = \sum_{\nu}^{N_\nu} q_\nu + q_{las}, \quad q_r = \left(\vec{j}\cdot\vec{E}\right), \tag{2}$$

$$q^\Sigma = q + q_e + q_e^h + q_i + q_i^h,$$

where U_ν is the volumetric density of the wideband radiation; c is the speed of light; $n = 0$ is a flat layer; $n = 1$ is an infinite one-dimensional cylinder; and $n = 2$ is a sphere. Boundary conditions for the diffusion approximation equations can be formulated on the external boundary from the lack of incoming radiation and the presence of an axis of symmetry. Note that the variables used in systems of equations (1) and (2) were determined according to the procedure in [20].

The equation for magnetic induction in the case of cylindrical symmetry (we assume below that in practice, only components $B_z \neq 0$, $E_\varphi \neq 0$, and ($B_\varphi = 0$, $B_r = 0$) are important):

$$\frac{\partial B_z}{\partial t} = \frac{c^2}{4\pi\mu J r}\frac{\partial}{\partial r}\left(\frac{J}{\sigma}\frac{\partial r B_z}{\partial r}\right).$$

It is evident that the system of equations must be supplemented by the corresponding initial and boundary conditions (e.g., the initial vorticities of the seed magnetic field).

Not that the direct acceleration of ions inside the plasma laser torch is negligible for density levels $q_{las} \leq (10^{20}–10^{23})$ W m^{-2} of the laser radiation flow, due to the current of the plasma's thermal electron conductivity compensating for the current of fast electrons .

In [20], we find methods for calculating energy Q_{ei} transferred per unit of time per unit volume from electrons to ions; the coefficients of the electron and ion thermal conductivity $\lambda_{e,i}^{\perp,\parallel}$ in the case of a magnetized plasma; optical $\chi_i(T, \rho)$ and thermodynamic parameters of the working environment; Coulomb logarithm $\ln \Lambda_{ei}$, which considers elections and ions interaction; $\nu_{ii,k}$ values that are the average frequencies of ion–ion collisions; the electroconductivity σ of plasma; and ways of calculating state of matter equations (based on the average charge concept), the electron gas (with allowance for gas degeneration and energy spent on ionization), and the coefficients of laser radiation absorption.

Moving grids that adapt to features of a solution are often used in numerically solving the quasi-ID equations of gas dynamics. This approach improves the accuracy of results using relatively rough calculation grids. The use of dynamically adaptive grids means we must write approximated plasma dynamic equations (1) in moving coordinates, i.e., transition from orthogonal coordinates x^α to arbitrary curvilinear coordinates q^α with allowance for the dependence of this transference on time t. The system of Euler equations with arbitrary curvilinear coordinates q^α in this case has the semi-divergent form

$$\frac{\partial J}{\partial t} = \frac{\partial V_{add}}{\partial q^\alpha}, \quad \frac{\partial}{\partial t}(J\rho) - V_{add}\frac{\partial}{\partial q^\alpha}(\rho) + \frac{\partial}{\partial q^\alpha}(\rho\upsilon^\alpha) = F_\rho.$$

$$\frac{\partial}{\partial t}(J\rho\upsilon^i) - V_{add}\frac{\partial}{\partial q^\alpha}(\rho\upsilon^i) + \frac{\partial}{\partial q^\alpha}(\rho\upsilon^\alpha\upsilon^i) + \frac{\partial p}{\partial q^\alpha} = F_{\rho\upsilon}, \tag{3}$$

$$\frac{\partial}{\partial t}(J\rho e) - V_{add}\frac{\partial}{\partial q^\alpha}(\rho e) + \frac{\partial}{\partial q^\alpha}\left(\rho\upsilon^\alpha e\right) + P\frac{\partial}{\partial q^i}(\upsilon^i) = F_E,$$

where P, ρ, and T are pressure, density and temperature; e is the gas's internal energy; $J = \dfrac{\partial x^\alpha}{\partial q^\alpha}$ is the Jacobian of transformation $x^\alpha = f\left(q^\alpha, t\right)$, υ^i is a component of the velocity vector; and V_{add} is the velocity of the adaptive system of coordinates. The gas–dynamic parameters associated with the adaptive movement of the grid are considered in this work by using the interpolation profile approach in [21–23].

NUMERICAL METHOD OF CALCULATING PLASMA PARAMETERS

Despite the one-dimensional character of the problem of solving gas dynamic equation (1–3), it imposes strict requirements on the numerical methods

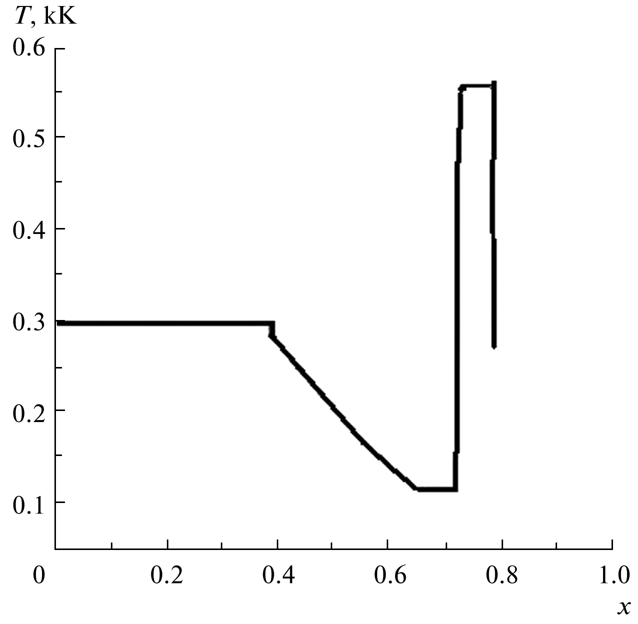

Fig. 1. Spatial distribution of the temperature T as for the time moment of $t = 3.31$mc obtained during solving of the Riemann problem (initial conditions are given in the text).

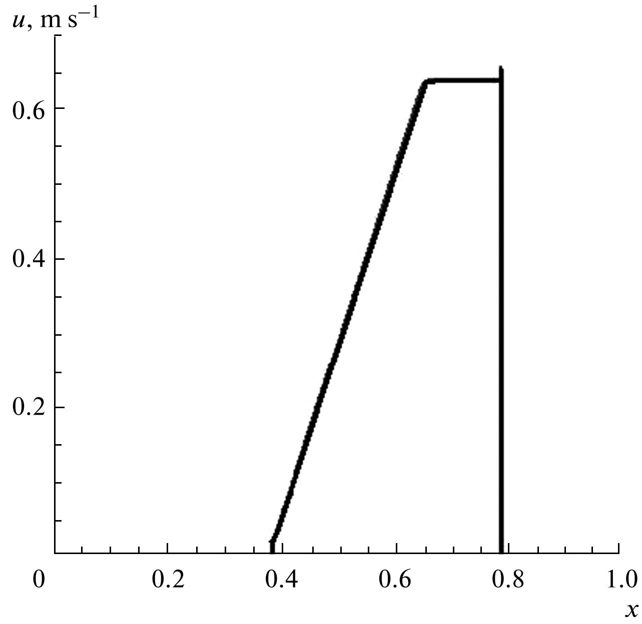

Fig. 2. Same as in Fig. 1 for the spatial distribution of velocity u.

used in solving it. The calculation scheme must first have improved dispersive and dissipative properties; it must be efficient and algorithmically simple; and it must have have monotonous properties and approximate smooth solutions, preferably with the maximum level of accuracy. The method for numerically solving the quasi-ID double temperature—one liquid equation of gas behavior that is based on the fractional steps method (in our case consisting of two steps) meets these requirements. Gas dynamic processes are considered at the first fractional step. The hyperbolic part of the system of equations (1—3) corresponds to these processes. The radiation transfer and electromagnetic processes in such devices are considered during the second fractional step.

METHODOLOGICAL AND TEST CALCULATIONS

Note that the above systems of differentiated equations relative to time variable t are the system of standard differentiated first-order equations that can be solved using the vector version of the multi-step Runge—Kutta method (a four-step variant with a fourth order of approximation according to time t was used in this work).

Numerical solutions to the equations for magnetic field diffusion and thermal conductivity were obtained using the compact difference scheme of improved accuracy in [24]. The method for calculating wide-band radiation transfer was considered on the basis of the multi-group diffusion approximation in [19]. The time step based on Δt that was needed for the integration of the above compact polynomial difference scheme was selected on the basis of the Courant—Friedrichs—Levi stability criteria.

A number of test (model) tasks must be performed for a substantiated numerical analysis of the plasma dynamic equations. The one-dimensional variant of the Riemann problem (with certain initial data, this task is sometimes referred to as the Soda problem [25] with adiabatic exponent $\gamma = 1.4$) associated with the breakdown of the unstable discontinuity of a given configuration is used as a test of the convective part of the problem. In this problem, it is considered that the area of integration is unit segment $x \in [0, 1]$ (coordinate x^{α} is standardized for the size of the calculated area) which is bifid ($x_{1/2} = 1/2$). The proper values of density ρ ($\rho_1 = 22.54$ kg m^{-3}; $x < 1/2$ and $\rho_2 = 0.1186$ kg m^{-3}; $x \geq 1/2$), pressure P ($P_1 = 19$ atm; $x < 1/2$ and $P_2 = 0.1$ atm; $x \geq 1/2$) and gas velocity $\nu = 0$ inside the area of calculation are set in every part at initial moment in time $t = 0$. The results from calculating these values depending on parameter x are given in Figs. 1—4 for time $t = 3.31$ms. The solution obtained with the more detailed grid was considered to be more accurate than the approximated solution. A comparison of the two showed that the difference was no more than one percent.

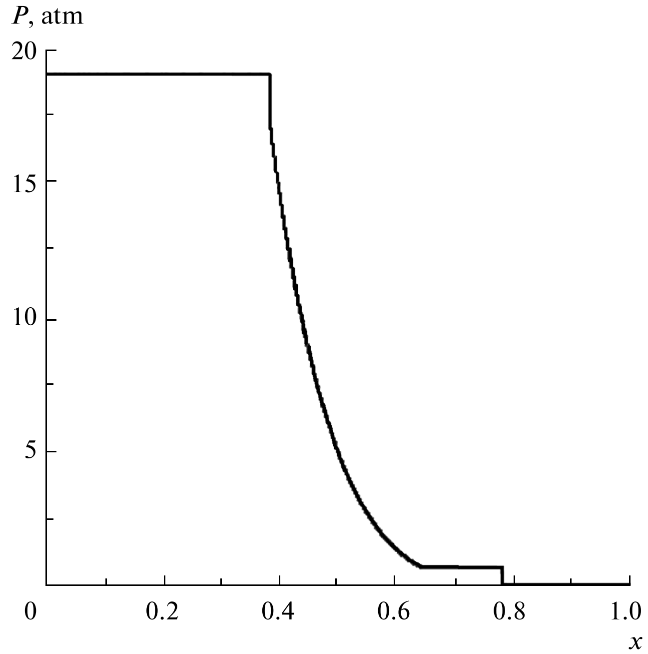

Fig. 3. Same as in Fig. 1 for the spatial distribution of pressure *P*.

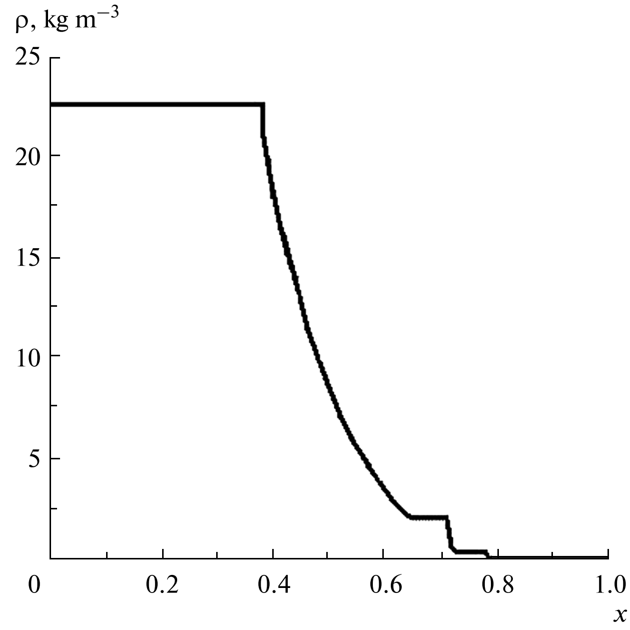

Fig.4. Same as in Fig. 1 for the spatial distribution of density velocity ρ.

CONCLUSIONS

A combined scheme of hot plasma confinement was proposed, and laser and plasma-based methods for generating a megagauss field during the implosion of a magnetized target were given. A mathematical model describing the physical processes in a magnetic–inertia system for hot plasma confinement was developed. A procedure for numerically calculating the main physical processes involved in a target plasma in an external magnetic field was presented. It was shown that the density, pressure, and gas velocity dependences inside the area of calculations and obtained using this method do not depend on the change in the dimensional reference grids.

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